INTERPOLATION AS AN IMPORTANT TOOL FOR

A. SPALVINS, J. SLANGENS,

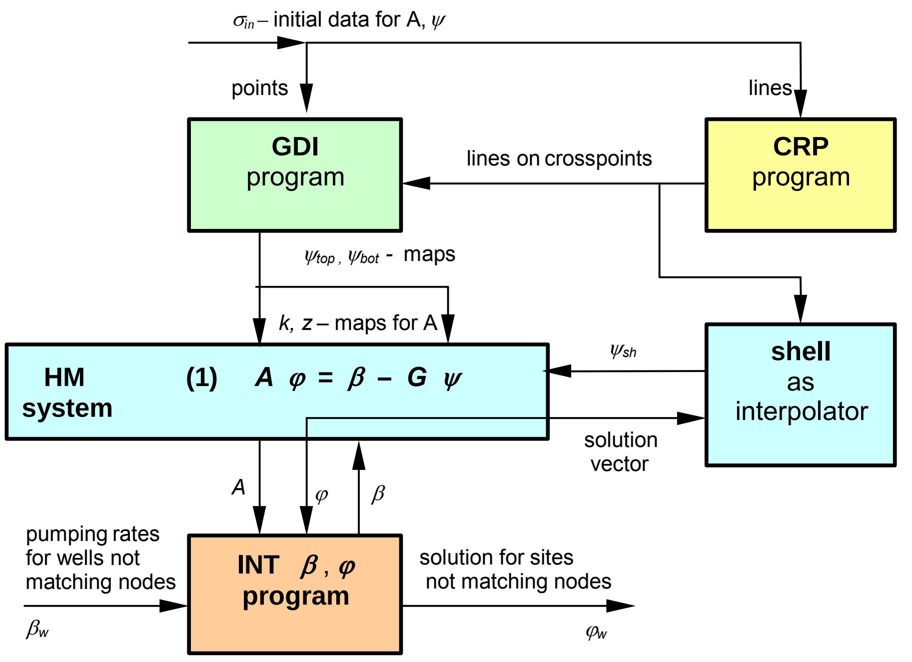
Riga Technical University, Environment Modelling Centre,

ABSTRACT

The quality of a hydrogeological model (HM) depends not only on credibility of initial data, but also upon interpolation technologies applied to create HM. The Environment Modelling Centre (EMC) of the Riga Technical University has developed reliable interpolation tools. Theoretical ideas implemented into them are explained.

INTRODUCTION

Algebraic equation system (1) for HM is specified on the *xyz*-grid built of ($h\times h\times m$)-sized blocks; *h* and *m* are the constant plane size and a variable height of blocks



φ - solution vector (groundwater head) at nodes of the HM grid; it may be necessary to interpolate $\varphi \to \varphi_w$, at sites not matching the nodes.

Primary elements *A*, β, ψ of (1) are obtained by interpolation:

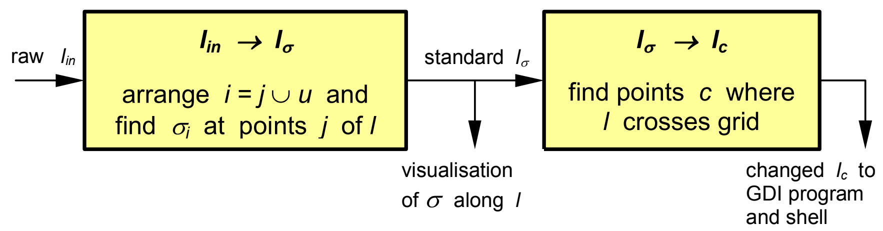
- *A* - symmetric matrix of the geological environment represented by a rectangular (2*s* + 1)-tiered *xy*-layer system where (*s* + 1) and *s* are the number of aquifers and interjacent aquitards; to obtain *A*, permeability *k* and elevation *z*-maps for each layer should be created.
- ψ - boundary head vector (Dirichlet's condition); ψ_{top}, ψ_{bot}, and ψ_{sh}-maps should be prepared for HM top z_0, bottom z_{2s+1} and shell (four vertical sides of HM) surfaces, correspondingly.
- *G* - diagonal matrix (part of *A*) assembled of elements linking the nodes where φ must be found with the ones where ψ is given.
- β - vector of water pumping rates (Neumann's boundary condition); to obtain β, interpolation $\beta_w \to \beta$ is needed.

For these interpolations, special tools have been developed (see the above scheme):

- geological data interpolation (GDI) program for creating the *k*, *z*, ψ_{top}, ψ_{bot}-maps (called σ-maps) and the program CRP (CRoss Point) for serving GDI and the shell;
- interpolation program INT β, φ for performing $\beta_w \to \beta$ and $\varphi \to \varphi_w$;
- to provide ψ_{sh}, the HM shell is acting as an interpolation device.

THE CRP PROGRAM

The CRP program turns a raw data line l_{in} into standard l_σ and forms $l_\sigma \to l_c$ for the GDI program and the shell. The data lines l_{in}, l_σ, l_c lines are based on *l*



Vectorized *l* passes master points *j* where its direction changes. The points *j*, *j + 1* are linked by the directed straight segment $l_{j,\,j+1}$, and *l* is the series:

$$l = \{\theta_j\} = \{x_j, y_j\},\quad j = 1, 2, \dots, J,\quad d_{j,\,j+1} = \sqrt{(x_j - x_{j+1})^2 + (y_j - y_{j+1})^2},\quad d_{1,\,J} = \sum_{j=1}^{J-1} d_{j,\,j+1}$$

where θ_j is coordinates of the *j*-th point; $d_{j,\,j+1}$ and $d_{1,\,J}$ are lengths of $l_{j,\,j+1}$ and *l*.

Raw $l_{in} = \{l, \sigma_{in}\}$, $\quad\sigma_{in} = \{\theta_u, \sigma_u\}$, $\quad u = 1, 2, \dots, U$; *u* and *j* may not coincide.

Standard $l_\sigma = \{\theta_i, \sigma_i\}$, $i = j \cup u = 1, 2, \dots$, and interpolation $\sigma_{in} \to \sigma_j$.

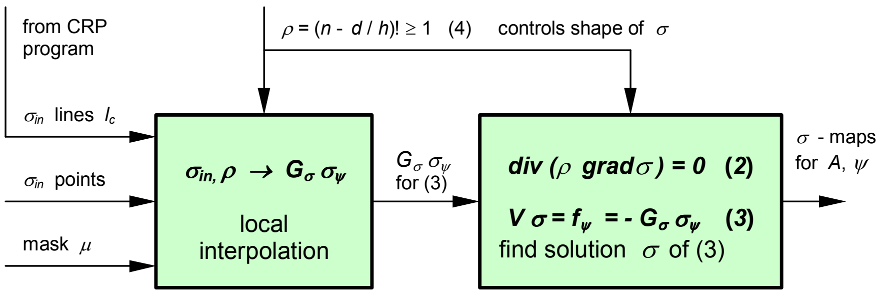
Changed $l_c = \{\theta_c, \sigma_c\}$, $c = 1, 2, \dots, C$, $d_{1,\,C} \to d_{1,\,J}$ if $h \to 0$; l_c is based on points *c* where *l* crosses the grid and σ_c are treated by the GDI program as an irregular part of the system (3).

In GDI, basic *l* is used for creating a mask μ. The line nullifies links v_{xy} of (3) crossed by *l*, and the *xy*-plane gets dissected into parts needed for μ.

Data lines including *j* and *c*-points are shown in the example demonstrating the GDI program.

THE GDI PROGRAM

The GDI program provides a σ-map as the numerical solution of the Laplace's boundary problem (2) for a heterogeneous environment



On the chosen *xy*-plane of (1), the algebraic equation system (3) approximates (2) where ρ is the factorial function (4) for positive rational numbers; *V* is the symmetric matrix of links $v_{xy} = \rho$; G_σ is the diagonal matrix (part of *V*) of elements linking the nodes, where σ must be found, with the σ_ψ-nodes of the Dirichlet's boundary condition σ_ψ interpolated from σ_{in}.

The shape of σ is controlled by ρ enclosing σ_ψ-nodes. In (4), *n* gives the radius $n\times h$ of the $\rho > 1$ area; *d* is the distance from the σ_ψ-node to the one where $\rho > 1$ must be specified. When $n = 1$, $\rho = 1.0$, and then peaks of σ may appear at the σ_ψ-nodes. These peaks may be turned into dome-shaped when $n \geq 4$. If necessary, ρ may be controlled for each σ_ψ-node, or along any line chosen. Weighty advantage of (3) is upholding maximum/minimum and minimal energy principles. This enables to apply minimal root sets of σ_{in}.

A round of GDI starts with local interpolation $\sigma_{in} \to \sigma_\psi$ that involves pointwise and line data (*e* and *c*-data). Hence the *e*-data has the lowest rank, they are processed first:

$$\sigma_0 = \sum_{i=1}^{t} C_i\,\sigma_i,\quad \sum_{i=1}^{t} C_i = 1.0,\qquad C_i = c_{i0} / \sum_{j=1}^{t} c_{j0},$$

$$c_{i0} = (1 - |\xi_i| / h)(1 - |\eta_i| / h),\qquad c_{i0} = 0\quad \text{if}\quad c_{i0} < 0.042. \tag{5}$$

The index *0* runs through *p = 1, 2, …, N* nodes of (3); σ_0 is found for the *0*–th node, as the centre of the search area L_σ bounded by hyperbolic arcs; C_i and c_{i0} are total and partial weights of the source σ_i; $\xi_i = x_i - x_0$, $\eta_i = y_i - y_0$ are local coordinates of σ_i. For an $h\times h$ block, (5) is illustrated by Fig. 1 and Fig. 2.

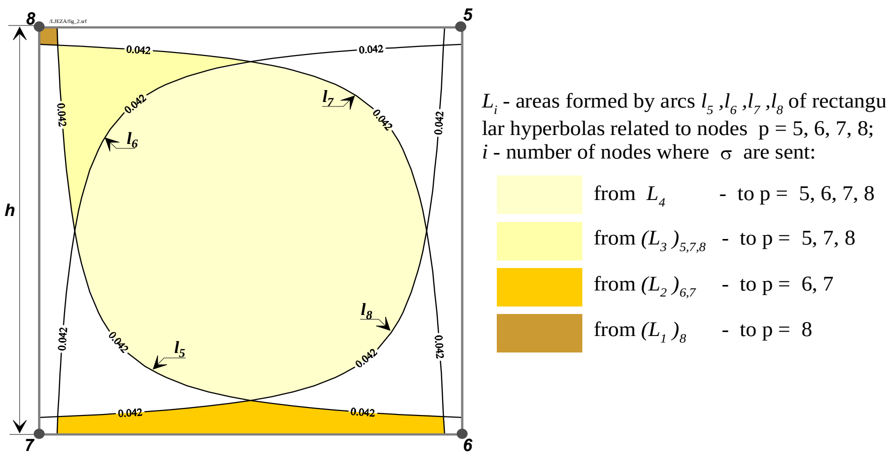


Fig. 1 Areas for point data search within an elementary $h\times h$ block if the optimal value λ = 0.042 (for the parameter of the hyperbolas-borderlines l_5, l_6, l_7, l_8) is applied.

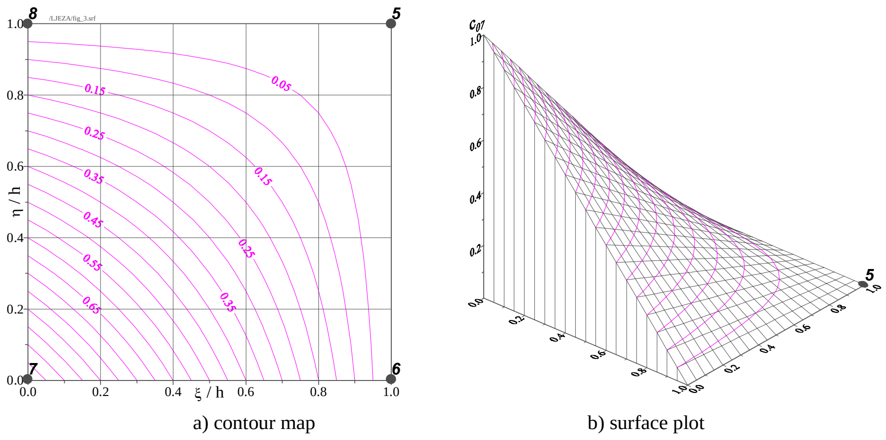
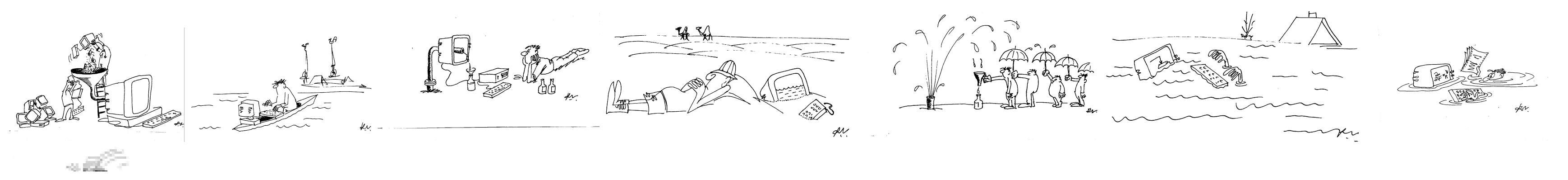


Fig. 2 Rectangular hyperbolas as the contours (isolines) of c_{07} on the grid block of Fig. 1.

The grid of (3) is controlled by the mask μ: if for the *p*-th node, $\mu_p = 1$ or *0* then σ_{in} are allowed or blocked here.

Commonly, *c*-data are carried by l_σ. For GDI, the CRP program finds l_c based on the *c*-points. Local *c*-interpolation eliminates them, thus providing f_ψ of (3) and annihilating the result of (5) there, because l_c has higher rank upon *e*-data. Local conflicts of l_σ by different ranks may also cause serious errors. These ranks can be accounted for by repeating several rounds of (3). This way is convenient for detecting possible errors, and much simpler σ_{in} may be applied than if one tries to obtain σ at once. The surface created by GDI may include sharp edges and rows specified by l_σ.

The GDI program in action is demonstrated here by an example of creating a ψ_{top}-map.

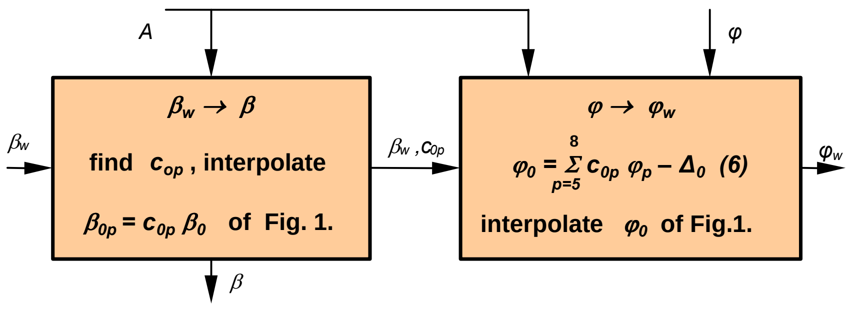


CREATING CREDIBLE HYDROGEOLOGICAL MODELS

R. JANBICKIS & I. LACE

1, k.4 Meza Street, Riga, LV-1048, Latvia; E-mail: emc@egle.cs.rtu.lv

THE INT β, φ PROGRAM



When A and ψ of (1) are ready, $\beta_w \rightarrow \beta$ ($\beta_0 \rightarrow \beta_{op}$) is interpolated $\beta_{op} = c_{op}\,\beta_0$ to nodes p = 5, 6, 7, 8 of the grid block (Fig. 1) by the weights c_{op}.

The back-interpolation $\varphi \rightarrow \varphi_w$, at the 0-th site of Fig. 1, applies the formula (6) where $\Delta_0 = \Sigma\, \lambda_{0j}\, \beta_j$ (j = 1,2, …, t) is the local depression caused by β_j sources via the weights λ_{0j}. Obtaining of c_{op} and λ_{0j} are explained in [1]. The principal element of λ_{0j} is the source function τ_0. Its contour map is shown in Fig. 3. The shape of τ_0 is close to a circle; in nodes, τ_0 = 0.

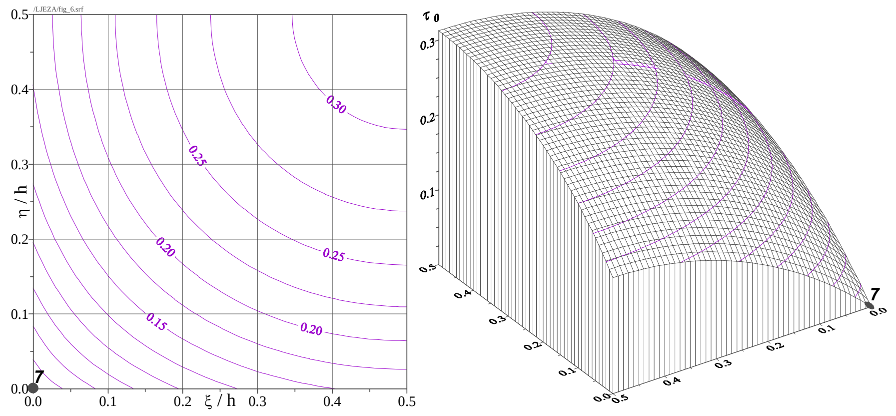


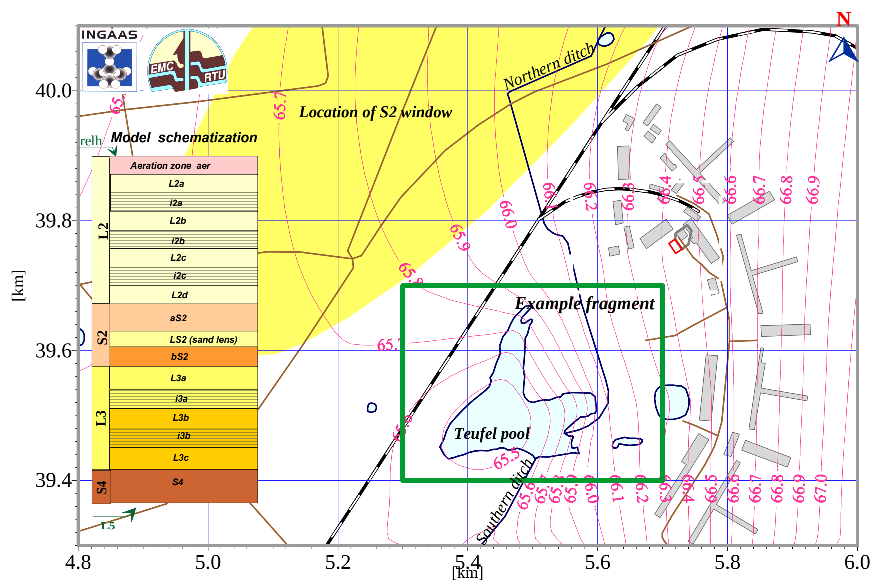
Fig. 3 Contours of τ_0 on the quarter of the elementary grid block.

THE SHELL OF HM AS AN INTERPOLATOR

In many cases, conventional software or modeller is helpless to provide the right ψ_{sh}-distribution. The problem has been solved by converting the shell into an interpolation device by enlarging (10^3 - 10^5)-fold values of its links. The shell then acts like an almost ideally conducting shield computing missing values of ψ_{sh}, as a special portion of φ where no ψ components are fixed. This method may be applied in all kinds of modelling programs developed for HM.

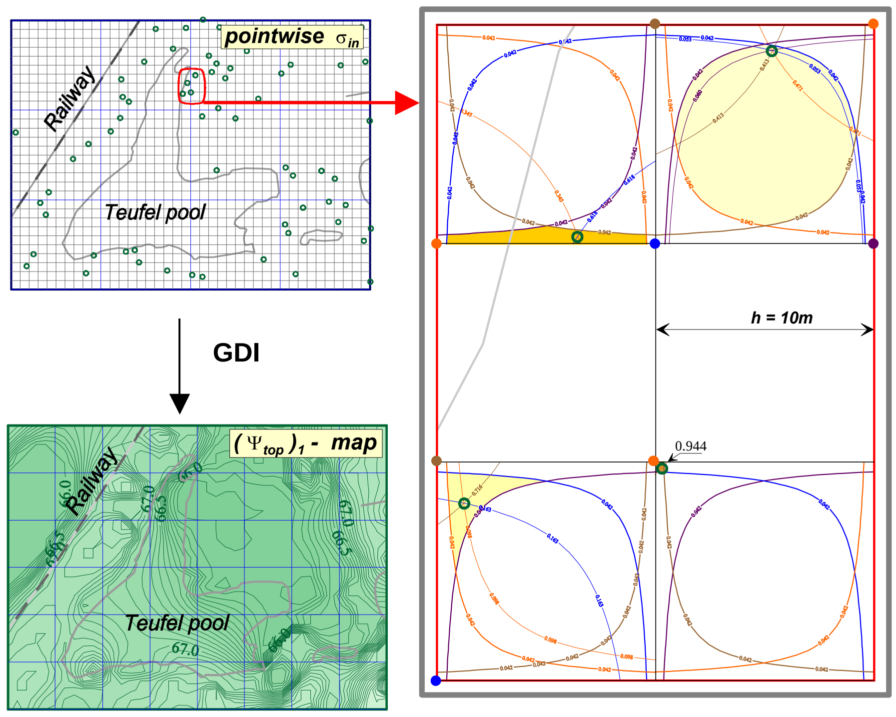
CREATING BY THE GDI PROGRAM A FRAGMENT OF THE ψ_{top}-MAP FOR BERNAU HM, GERMANY

Materials of Bernau HM are taken from [2]. The example ψ_{top}-map presents the ground surface elevations with water pools included. The map is created in two rounds. For the first round, only pointwise σ_{in} of ground elevations are applied. The intermediate result $(\psi_{top})_1$ bears no impact of the pools and the railway embankment. These objects are accounted for during the second round by using line σ_{in}.



Undisturbed piezometric head [m asl] distribution for the ***L2a*** aquifer with Bernau HM schematization shown

The first round of the GDI program



local interpolation (5) for σ_{in}

The second (final) round of the GDI program

Pointwise $(\psi_{top})_1$-map is masked to clear areas of pools where their banklines specify water elevations of pools as solution of (3). Railway line itself clears its way through pointwise σ_{in}.

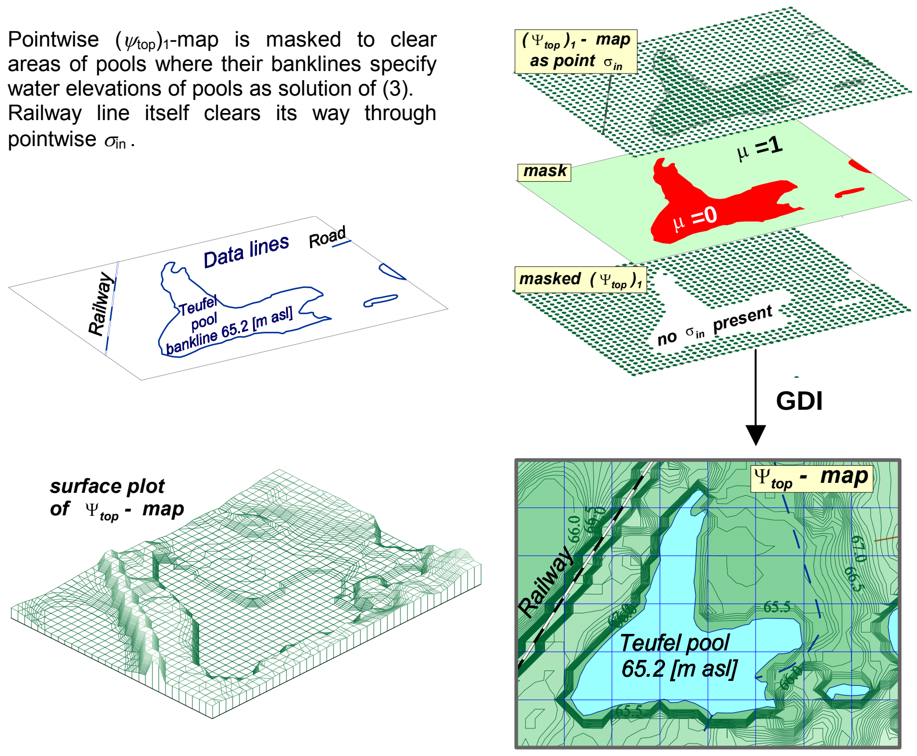
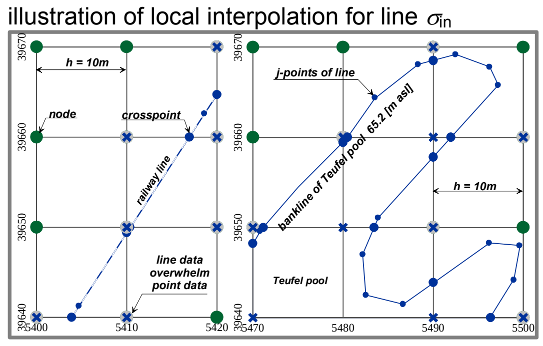


illustration of local interpolation for line σ_{in}



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[1] Proceedings of ModelCARE 2002, Prague, Czech Republic, 17-20 June, 2002.

[2] http://www.rtu.lv/www_emc/issue_43.pdf/as_bern.pdf

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NEW THEORETICAL IDEAS IMPLEMENTED

1. Method of local interpolation for pointwise data.
2. To apply crosspoints of data lines as a part of the HM system.
3. Using numerical solutions of boundary field problems for creating σ-maps by controlling the heterogeneousity parametre ρ.
4. Complex σ–maps are obtained gradually by repeating interpolation rounds.
5. Back-interpolation ($\varphi \rightarrow \varphi_w$) for irregular points of the HM body.
6. Using the HM shell as an interpolation device.