Landscape elevation maps as reliable boundary condition for hydrogeological models

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ABSTRACT: The team of the Environment Modelling Centre (EMC) of the Riga Technical University has developed an effective method incorporating the landscape elevation map, as the boundary condition of the three-dimensional (3D) hydrogeological model (HM). Due to this handy application, reliability of any HM improves considerably.

1 INTRODUCTION

To narrow our study, only semi-3D steady state HM, describing mean annual conditions, is considered. The *xyz*-grid of HM is built of ($h*h*h_z$)-sized blocks (h is the block plane size; h_z is a variable block height). They constitute a rectangular *xy*-layer system. Its four vertical sides compose the shell of HM. The ground surface *rel* and the lower side of the model are its geometrical top and bottom, accordingly. In HM, the vector φ of the piezometric head is approximated, in nodes of the 3D grid of HM, by the following algebraic equation system:

$$A\ \varphi = \beta - G\ \psi, \qquad A = A_{xy} + A_z - G, \qquad \beta = \beta_{in} + \beta_{bot} + \beta_{sh} + \beta_w, \qquad \beta_\psi = G\ (\psi - \varphi) \tag{1}$$

where the matrices A_{xy}, A_z, G represent, correspondingly, the horizontal links of aquifers, the vertical ties originated by aquitards, the elements connecting nodes of the grid with the piezometric boundary conditions ψ; the vector β accounts for the fixed boundary flows: β_w is the given water production rate in wells; β_{in}, β_{bot} and β_{sh} are the boundary surface flows, which may be specified on the top, bottom and shell areas of HM, correspondingly; β_ψ is the computed flow passing through elements of G.

Unluckily, true distributions of the flows β_{in}, β_{bot} and β_{sh} can hardly be obtained from field data. Crude substitutes of these fixed flows inevitably produce bad HM results. Fortunately, all three flows can be changed for the more exact ones of the β_ψ - type (Spalvins et al., 2000).This paper explains how the transformation $\beta_{in} \rightarrow \beta_{\psi\ in}$ of this flow is performed for the infiltration flow β_{in}, which dominates (1).

2 THE LANDSCAPE ELEVATION MAP AS THE BOUNDARY CONDITION

The EMC team has successfully applied (since 1995) the landscape elevation map ψ_{rel}, as the ψ-type boundary condition for (1). This method assumes that the ψ_{rel} surface is piezometric, and the aeration zone *aer* (part of unconfined aquifer q) behaves like an aquitard, according to the capillary van Genuchten's (VG) model for the unsaturated soil (Genuchten, 1980). The GV model predicts the distinct permeability values: $k_{aer} \sim 10^{-2}$ - 10^{-3} [m/day] and 1 - 10 [m/day], accordingly, for the areas of descending (infiltration) and ascending (discharge) flows of the aeration zone. The ascending flows are caused mostly by elements of the hydrographical network (rivers, lakes etc.) included in ψ_{rel}. Through the aeration zone, passes the flow $\beta_{\psi\ in}$ (automatic replacement of β_{in}):

$$\beta_{\psi\ in} = G_{aer}\ (\psi_{rel} - \varphi_q), \qquad g_{aer} = k_{aer}\, h^2 / h_{aer} \geq 0 \tag{2}$$

where φ_q is the computed head (subvector of φ) for q; G_{aer} (diagonal submatrix of G) contains the vertical ties g_{aer} connecting ψ_{rel} with φ_q; h_{aer} is the thickness distribution of the *aer* zone. The expression (2) reflects the usual support of HM, when the ψ-condition is applied. The $\beta_{\psi\ in}$-distribution is also helpful for the HM calibration - both, as its tool (g_{aer} variable) and target (some data for β_{in} and φ_q are available). The calibrated representation (2) must be achieved iteratively under the following important limitation used by EMC: "due to complexity of any geometrical change of HM, the geometry must remain fixed, until the final calibrated φ-distribution is obtained". Really, the thicknesses h_{aer} and h_q (saturated part of q) are variable, and this feature is accounted for by the current EMC methodology discussed below.

The aquifer q is supposed to be conditionally confined and the start try for its transmitivity distribution a_q is obtained, as follows:

$$a_q^{(0)} = c_q^{(0)}\, k_q^{(0)}\, h_q^{(0)}, \qquad h_q^{(0)} = \psi_{rel} - z_q + \Delta_q, \qquad \Delta_q = 2\ cm \tag{3}$$

where $c_q^{(0)}$, $k_q^{(0)}$, $h_q^{(0)}$ are the start distributions (represented by diagonal matrices), respectively, of the correction factor, permeability, thickness for q; z_q is the bottom surface of q; the constant Δ_q is used if q is discontinuous(in some areas, $\psi_{rel} - z_q = 0$); the distribution $h_q^{(0)}$ will remain unchanged, until the final calibrated results will appear. Usually, $c_q^{(0)} = 1.0$.

The HM geometry is not distorted by the small fictitious thickness Δ_q introduced to perform two important tasks, for the non-existent parts of q:

- to control a_q, if necessary, via the proper choice of k_q:
- to exclude any influence of computational rounding errors carried by ψ_{rel} and z_q involved, in the h_q calculations (3).

The EMC team applies the similar additional thickness $\Delta = 2\ cm$ for all kinds of discontinuous or fictitious geological layers put in action of HM.

For g_{aer}, the formula (2) must be specified:

$$g_{aer}^{(0)} = c_{aer}^{(0)}\, k_{aer}^{(0)}\, h^2 / h_{aer}^{(0)}, \qquad h_{aer}^{(0)} = \Delta_{aer} = 2\ cm\ . \tag{4}$$

For the *aer* zone, the meaning of $c_{aer}^{(0)}$, $k_{aer}^{(0)}$ and $h_{aer}^{(0)}$ is the same, as for the similar distributions of q. The start thickness Δ_{aer} (like $h_q^{(0)}$) will be kept constant, in order not to distort the initial geometry of HM where the real, unknown h_{aer}-distribution is not accounted for. The $k_{aer}^{(0)}$-distribution is obtained, as follows: the district values $k_{aer} = 10^{-3}$ and 1.0 are applied, correspondingly, as the areal base for the k_{aer}-map and for the lines (or areas) of the hydrographical network. Usually, $c_{aer}^{(0)} = 10^{-2}$, if the mean thickness of the aeration zone is assumed ~ 2 metres.

When the distributions (3) and (4) are applied, HM naturally provides good results, especially for φ_q, because they are governed by ψ_{rel}, as follows:

- the φ_q surface automatically reflects main features of ψ_{rel}, because these surfaces are interlinked via g_{aer}; this phenomenon always takes place in nature;
- HM provides for a modeller the $\beta_{\psi\ in}$ distribution of (2); it informs about the intensity of infiltration and also clearly indicates areas of discharge and recharge for the q aquifer;

The complex distribution $\beta_{\psi\ in}$ can be controlled via simple change of g_{aer}. The expected response of $\beta_{\psi\ in}$ is easy to predict. It follows from the above considerations that the advantage of $\beta_{\psi\ in}$ (predicted by HM) over conventional β_{in} (masterminded by a modeller) is enormous, because $\beta_{\psi\ in}$ is based on reliable initial data (carried mostly by ψ_{rel}) used by (2).

After the initial calibration of HM ($k_q^{(0)} \rightarrow k_q^{(1)}$; $k_{aer}^{(0)} \rightarrow k_{aer}^{(1)}$), the solution $\varphi_q^{(1)}$ is obtained, and the improved distributions $h_{aer}^{(1)}$ and $h_q^{(1)}$ can be specified:

$$h_{aer}^{(1)} = \begin{cases} \delta^{(1)} = \psi_{rel} - \varphi_q^{(1)} & if\ \ \delta^{(1)} \geq \Delta_{aer}, \\ \Delta_{aer} & if\ \ \delta^{(1)} \leq \Delta_{aer}, \end{cases} \tag{5}$$

$$h_q^{(1)} = h_q^{(0)} - h_{aer}^{(1)} + \Delta_{aer}\ . \tag{6}$$

In (5), $\delta^{(1)} \leq \Delta_{aer}$ holds mostly for the discharge areas where real h_{aer} is rarely known. For this reason, there is applied $h_{aer}^{(1)} = \Delta_{aer}$.

Hence the distributions $h_{aer}^{(1)}$, $h_q^{(1)}$ cannot be applied directly (HM geometry fixed temporarily!), the correction factors $c_q^{(1)}$, $c_{aer}^{(1)}$ must be computed:

$$c_q^{(1)} = h_q^{(1)} / h_q^{(0)}, \tag{7}$$

$$c_{aer}^{(1)} = \begin{cases} \sigma^{(1)} = 2\, c_{aer}^{(0)} / h_{aer}^{(1)} & if\, \sigma^{(1)} \le c_{aer}^{(0)}, \\ c_{aer}^{(0)} & if\, \sigma^{(1)} > c_{aer}^{(0)}. \end{cases} \tag{8}$$

For $\sigma^{(1)} > c_{aer}^{(0)}$, the empiric rule (8) prevents from sharp increasing of these components of $\beta_{\psi\ in}$,which are located in areas where $h_{aer}^{(1)} < 2$ metres.
The factors $c_q^{(1)}$, $c_{aer}^{(1)}$ must be applied for the next approximation of the values a_q and g_{aer} :

$$a_q^{(1)} = c_q^{(1)}\, k_q^{(1)}\, h_q^{(0)}, \qquad g_{aer}^{(1)} = c_{aer}^{(1)}\, k_{aer}^{(1)}\, h^2 / \Delta_{aer} \quad . \tag{9}$$

If after the *i*-th iteration, the final calibrated result $\varphi_q^{(i)}$ is obtained then the HM geometry can changed, in accordance with the following representation:

$$a_q^{(i)} = c_q^{(i)}\, k_q^{(i)}\, h_q^{(0)} = k_q^{(i)}\, h_q^{(i)}, \tag{11}$$

$$g_{aer}^{(i)} = c_{aer}^{(i)}\, k_{aer}^{(i)}\, h^2 / \Delta_{aer}.= k_a^{(i)}\, h^2 / h_{aer}^{(i)}, \quad k_a^{(i)} = c_{aer}^{(i)}\, k_{aer}^{(i)}\, h_{aer}^{(i)} / \Delta_{aer}. \tag{12}$$

The real HM geometry is important when contaminant mass transport problems are to be investigated. In practice, even the crude start attempts (3) and (4) provide surprisingly good results for complex regional HM (Spalvins et al., 1996; Gosk et al., 1999). In these cases, likely, no results of HM can be obtained if the conventional approach of fixed β_{in} is applied.
The above method can be run by any HM system, for example, by the Groundwater Vistas program (Environmental Simulations, 1997). However, no regimes of unconfined aquifers must be used there, because the MODFLOW code involved may ruin HM, especially, if HM contains discontinuous layers. Then the destruction is inevitable, due to irreparable, automatic annihilation of "dried" (saturated thickness equals zero) cells of HM. This cannot be prevented for the non-existent areas of discontinuous aquifers, because their thickness is zero, as the geometrical feature.
To immerse the ψ_{rel}-map, in the conventional MODFLOW code environment, the surface *rel* should be specified, as a formal thin aquifer (Δ_{rel} = 2 cm).

3 CONCLUSIONS

The infiltration flow, on the HM top, is the dominant boundary condition. Unfortunately, for complex regional HM, the conventional way of using fixed boundary flows cannot provide good results.
The EMC team has developed and proved the new method of obtaining this boundary flow via using the landscape elevation map, as the reliable piezometric boundary condition. The method can be run by any modelling system related to creating of HM.
Reliability of HM results has increased drastically, even for the case, when the crudest form of the new method is applied.

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Spalviņš A. Zemes virsmas augstuma kartes kā drošs robežnoteikums hidroģeoloģiskajam modelim.
Rīgas Tehniskās universitātes Vides Modelēšanas centra (VMC) grupa ir radījusi efektīvu metodi, kura izmanto virsmas augstuma karti kā robežnoteikumu trīsdimensiju (3D) hidroģeoloģiskajā modelī (HM). Pateicoties šim veiksmīgajam paņēmienam, jebkura HM drošums tiek būtiski uzlabots.

Спалвинь А. Карта высот поверхности земли как надежное граничное условие гидрогеологической модели.
Группой Центра моделирования окружающей среды (ЦМОС) Рижского Технического университета разработан эффективный метод, который применяет карту высот поверхности земли, как граничное условие для трехмерной (3D) гидрогеологической модели (ГМ). Благодаря этому удачному подходу, надежность любой ГМ заметно улучшается.