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Beam propagation in non-linear tapered co-ordinate system

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ABSTRACT: A beam propagation technique is presented which allows the solution of the wave equation in the case of a dielectric taper without the necessity of staircasing. The method consists in application of the non-linear tapered co-ordinate system. As a result curved boundaries between core and cladding can be modelled exactly. The results obtained are compared with the ones computed using the standard Beam Propagation algorithm in the rectangular and tapered co-ordinate system. In all analysed cases the novel approach: Non-linear Tapered Beam Propagation Method allows calculating the field distribution and the field overlap at the end of the structure faster and using less computer memory.

1 INTRODUCTION

Integrated optics has many applications in the field of telecommunications. Due to fast technological progress ever more complicated devices can be fabricated for the design and optimisation of which fast and accurate software tools are needed.

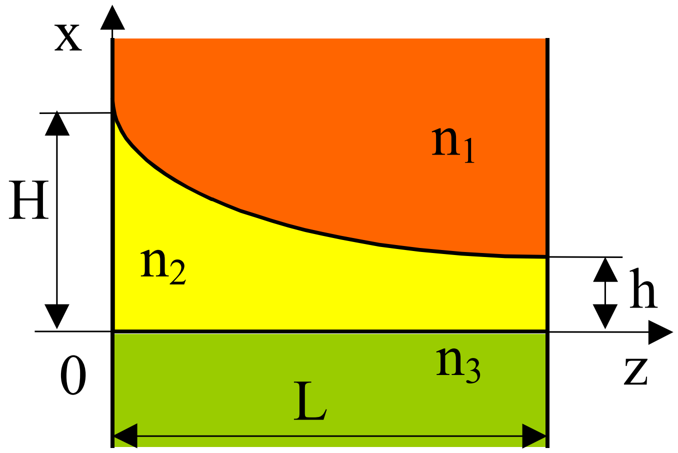


Fig.1. Asymmetric 2D non-linear taper structure.

One of the commonly used structures in integrated optical circuits is the optical taper (Fig.1). The optical tapers are used for example as spot size transformers or in multimode couplers and tapered lasers (Snyder & Love 1987, Ebeling 1993, Kasaya at. al. 1993, Lorenzo at. al. 1998). The standard technique used for the design and optimisation of these devices is the Beam Propagation Method (BPM) (Yevick 1994, März 1995). Usually BPM is applied in the rectangular co-ordinate system. As a result the oblique boundaries between core and cladding cannot be modelled exactly and are approximated using staircasing. Consequently an unphysical numerical noise resulting from the presence of dielectric discontinuities, which is observed in the field plots, influences negatively the accuracy and increases the computer memory requirement as well as calculation time (Sujecki at. al. 1999). In the case of the linear taper this problem can be avoided using the tapered co-ordinate system (Sewell at. al. 1996, Sujecki at. al. 1999). This approach was also extended to general non-linear taper structures using local linearisation (Sujecki at. al. 2000). However, it is found that further improvement of the algorithm efficiency can be achieved by utilisation of a suitable co-ordinate system, namely non-linear tapered co-ordinate system.

The non-linear tapered co-ordinate system allows accurate modelling of curve-linear interfaces between the core and cladding. It is shown that as a result faster convergence rate both with transverse and longitudinal step can be achieved, than in the case of the standard rectangular and tapered BPM. Hence accurate results can be obtained faster and using less memory.

2 FORMULATION

The scalar (TE) wave equation in the rectangular co-ordinate system x, y and z has the form:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}+k^2(x,y,z)\right)\psi(x,z)=0 \quad (1)$$

where $k = n(x,z)\, 2\pi/\lambda_o$, $n(x, z)$ is the refractive index (Fig.1) and $\psi(x,y,z)$ denotes the y component of the electric field.

The non-linear tapered co-ordinates u and w are introduced as:

$$x = f(u,w) \quad \text{and} \quad z = w$$

where f(u,w) is a suitably chosen function, so that one of the lines $x = f(u = \text{const}, z)$ coincides with the curved boundary between core and cladding (Fig.1).

The derivatives appearing in (1) can be then expressed in the new co-ordinate system as:

$$\frac{\partial}{\partial x}=\frac{1}{\frac{\partial f}{\partial u}}\frac{\partial}{\partial u} \quad (2)$$

$$\frac{\partial}{\partial z}=\frac{\partial}{\partial w}-\frac{\frac{\partial f}{\partial w}}{\frac{\partial f}{\partial u}}\frac{\partial}{\partial u}$$

Substituting (2) into (1) and rearranging (Sujecki in press) gives the one-way wave equation in the non-linear tapered co-ordinate system:

$$\left(\frac{\partial}{\partial w}-\frac{\frac{\partial f}{\partial w}}{\frac{\partial f}{\partial u}}\frac{\partial}{\partial u}\right)\psi(u,w)=\pm j\sqrt{\beta^2+L}\;\psi(u,w) \quad (3)$$

where the operator L:

$$L=\frac{1}{\left(\frac{\partial f}{\partial u}\right)^2}\frac{\partial^2}{\partial u^2}+k^2-\beta^2$$

and β is the reference propagation constant.

The paraxial BPM can be now obtained using a truncated Taylor series expansion of the square root operator. After having introduced the envelope function - $\psi(u,z)=\varphi(u,z)\exp(\pm j\beta z)$ the paraxial wave equation in the non-linear tapered co-ordinate system is obtained:

$$\frac{\partial}{\partial w}\varphi(u,w)=\left(\frac{\frac{\partial f}{\partial w}}{\frac{\partial f}{\partial u}}\frac{\partial}{\partial u}\pm j\frac{1}{2\beta}L\right)\varphi(u,w) \quad (4)$$

The right hand side (RHS) operator in (4) can now be discretised using Finite Difference (FD). method. The resulting matrix equation can be solved using the Crank-Nicholson scheme and a standard tridiagonal matrix solver . At the boundary of the analysis window a transparent boundary condition is assumed (Hadley 1991).

3 RESULTS AND DISCUSSION

In order to compare the convergence rate of the Finite Difference Beam Propagation Method in the non-linear co-ordinate system (non-linear tapered FD-BPM) with the standard approaches, namely standard tapered FD-BPM and rectangular FD-BPM [10] the following structures are studied: an asymmetric air clad taper which is tapered down from a width of 0.8 μm to 0.4 μm over the length. - L equal to 22.9 μm and a semiconductor clad taper which is tapered from the width of 0.2 μm to 0.1 μm over a distance of either 5.73 μm or 57.3 μm. For the purpose of comparison an exponential taper profile is chosen, i.e. with the boundary between the core and superstrate depending on z according to the formula: $x = H*\exp(-p*z)$, were p is obtained from the condition $x(L) = h$. Consequently $f(u,w) = u*\exp(-w*p)$, $\partial f/\partial u = \exp(-w*p)$ and $\partial f/\partial w = -u*p*\exp(-w*p)$.

Table 1: Power content in the fundamental TE mode at the and of the tapers:

Method	P		
	Structure 1*	Structure 2 **	Structure 3 ***
Standard rectangular FD-BPM	0.9117	0.9728	0.8362
Standard tapered FD-BPM	0.9117	0.9729	0.8362
Non-linear tapered FD-BPM	0.9117	0.9729	0.8362

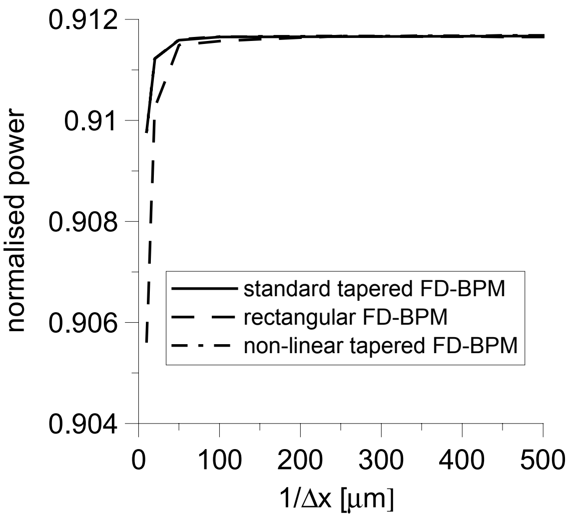
* Structure1: $W_i = 0.2$ μm, $W_o = 0.1$ μm, $n_f = 3.3$, $n_s = 3.17$, $n_c = 3.17$, L=5,73μm,
** Structure2: $W_i = 0.2$ μm, $W_o = 0.1$ μm, $n_f = 3.3$, $n_s = 3.17$, $n_c = 3.17$, L=57.3μm
*** Structure3: $W_i = 0.8$ μm, $W_o = 0.4$ μm, $n_f = 3.3$, $n_s = 3.17$, $n_c = 1.0$, L=22,9μm

In the Figs 2a ÷ 2c the dependence of the power content guided in the fundamental TE mode measured at the end of the taper on the transverse mesh size is shown. The initial field distribution corresponds to the local fundamental mode. As can be seen the results obtained applying the Beam Propagation Method in the non-linear tapered co-ordinate system (non-linear tapered BPM) converge much faster than in the case of the Beam Propagation Method in the rectangular co-ordinate system (rectangular BPM). In comparison with the standard tapered FD-BPM the convergence rate does not change.

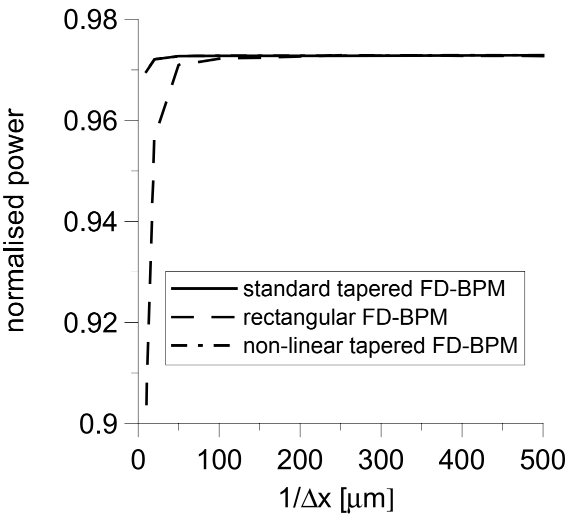
In the Figs 3a ÷ 3c the dependence of the power content guided in the fundamental TE mode measured at the end of the taper on the transverse mesh size is shown. It is noticed that the convergence rate in the case of the non-linear tapered FD-BPM is faster than in the case of standard tapered FD-BPM and comparable with the one obtained for the rectangular FD-BPM.

In the Fig. 4 the field intensity distributions are given in the case of the air clad taper of the length of 22.9 μm. The results presented are normalised with respect to the maximum field intensity of the initial field distribution. The isolines are given starting at 0.1 of the maximum value with an increment of 0.2. The results obtained confirm that the non-linear and standard tapered BPM

a)



b)



c)

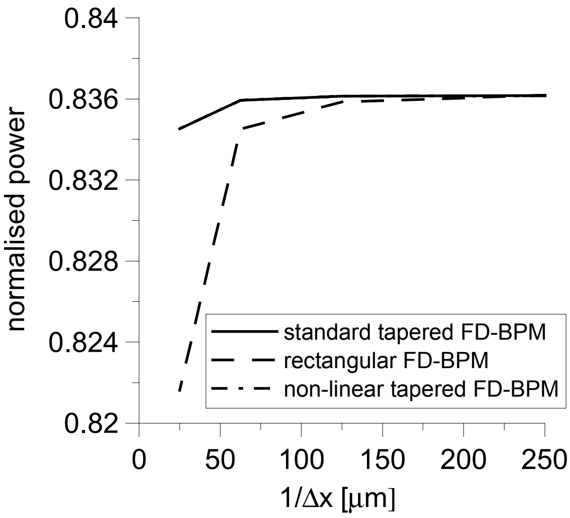


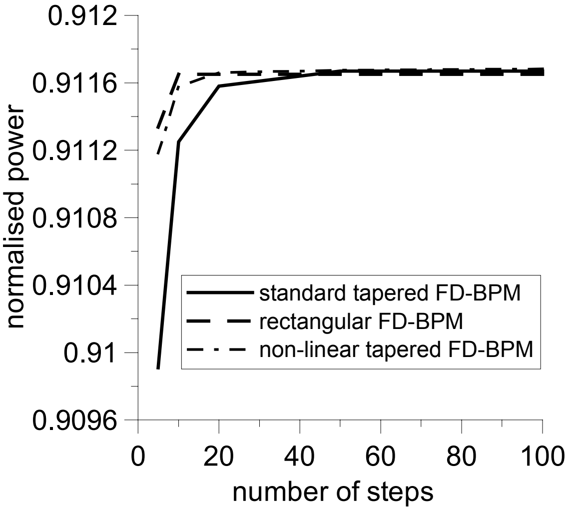
Fig.2. Dependence of the power content in the fundamental x-polarised mode at the end of a symmetrical taper on the transverse mesh size.

a) $H = 0.2$ μm, $h = 0.1$ μm, $L = 5.73$ μm, $n_1 = 3.17$, $n_2 = 3.3$, $n_3 = 3.17$, $\lambda = 1.55$ μm

b) $H = 0.2$ μm, $h = 0.1$ μm, $L = 57.3$ μm, $n_1 = 3.17$, $n_2 = 3.3$, $n_3 = 3.17$, $\lambda = 1.55$ μm

c) $H = 0.8$ μm, $h = 0.4$ μm, $L = 22.9$ μm, $n_1 = 1$, $n_2 = 3.3$, $n_3 = 3.17$, $\lambda = 1.55$ μm

a)



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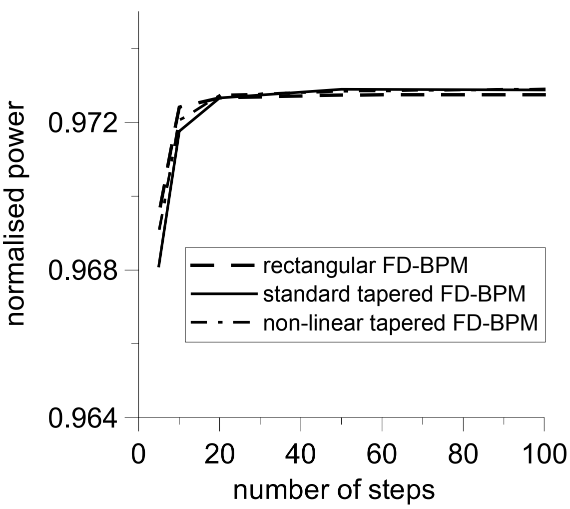

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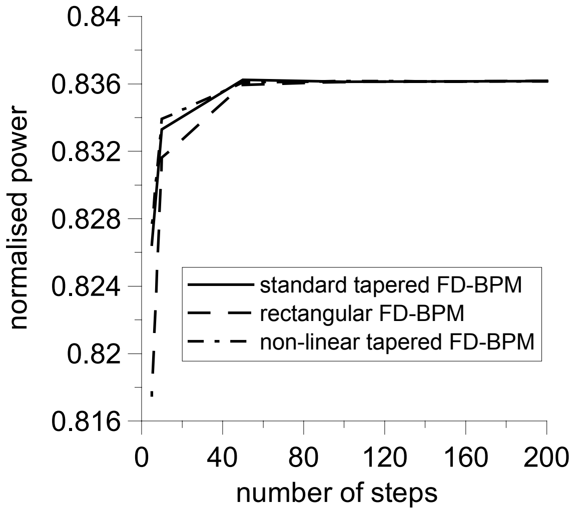


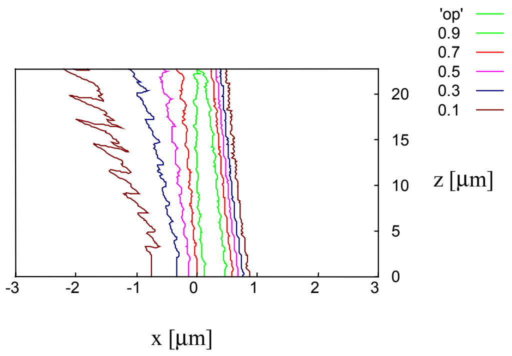
Fig.3. Dependence of the power content in the fundamental x-polarised mode at the end of a symmetrical taper on the number of the propagation steps.

a) H = 0.2 μm, h = 0.1 μm, L = 5.73 μm, n_1 = 3.17, n_2 = 3.3, n_3 = 3.17, λ = 1.55 μm

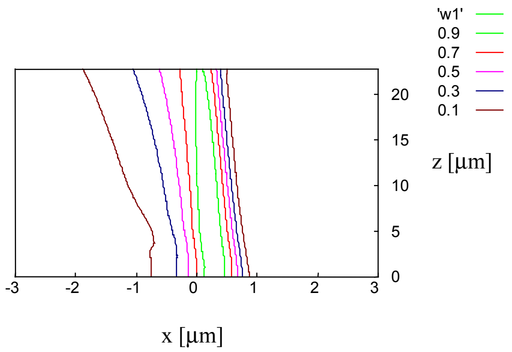
b) H = 0.2 μm, h = 0.1 μm, L = 57.3 μm, n_1 = 3.17, n_2 = 3.3, n_3 = 3.17, λ = 1.55 μm

c) H = 0.8 μm, h = 0.4 μm, L = 22.9 μm, n_1 = 1, n_2 = 3.3, n_3 = 3.17, λ = 1.55 μm

a)



b)



c)

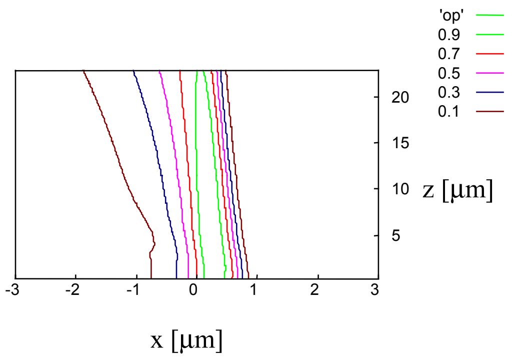


Fig.4. Field intensity plots for x-polarised case in symmetrical tapered structure.
H = 0.8 µm, h = 0.4 µm, L = 22.9 µm, $n_1 = 1$, $n_2 = 3.3$, $n_3 = 3.17$, $\lambda = 1.55$ µm

a) Rectangular FD-BPM
b) Standard tapered FD-BPM
c) Non-linear tapered FD-BPM

suppresses the numerical noise much more effectively than the standard rectangular BPM.

In the Table 1 the power content in the fundamental TE mode measured at the end of the taper for the structures studied is given. It is observed that the results obtained by the non-linear tapered FD-BPM and standard tapered and rectangular FD-BPM agree almost to the fourth decimal place.

4 CONCLUSIONS

The new technique: Beam Propagation Method in the non-linear tapered co-ordinate system is compared with the standard Beam Propagation algorithm in the rectangular and tapered co-ordinate systems in the case of a non-linear taper structure. The results obtained confirm that the new algorithm is faster and requires less computer memory to achieve a given degree of accuracy.

A very good agreement is also observed between the results calculated by the three independent Beam Propagation methods, namely non-linear tapered FD-BPM and standard tapered and rectangular FD-BPM.

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Sujecki S. Stara izplatīšanās nelineārā koniskā koordināšu sistēmā.
Tiek apskatīta stara izplatīšanās apraksta metode, kura ļauj atrisināt viļņu vienādojumu dialektiskā konusā bez kāpņveida aproksimācijas izmantošanas. Šī metode pielieto nelineāru konisko koordinātu sistēmu, kura ļauj precīzi modelēt liektās robežas starp serdeni un apšuvumu. Iegūtie rezultāti tika salīdzināti ar aprēķiniem, kuri veikti ar standartveida Viļņa Izplatīšanās algoritmu taisnleņķa un koniskās koordinātu sistēmās. Visos analizētajos gadījumos jaunā pieeja - Nelineārā Koniskā Stara Izplatīšanās Metode pieļāva ātrāk un ar mazāku datora atmiņas patēriņu aprēķināt lauka sadalījumu un tā pārlaidumu struktūras beigās.

Суецки С. Распространение пучка в нелинейной конической системе координат.
Рассматривается метод представления распространения пучка, который позволяет решить волновое уравнение в диалектическом конусе без использования аппроксимации лестничного типа. Этот метод применяет нелинейную коническую систему координат, которая позволяет точно моделировать криволинейные границы между сердечником и оболочкой. Полученные результаты были сопоставлены с вычислениями, которые выпольнялись стандартным алгоритмом Распространения Пучка в прямоугольной и конической системах координат. Во всех рассмотренных случаях новый подход - Метод Распространения Нелинейного Конического пучка дал возможность вычислить быстрее и с меньшими затратами памяти компютера распространение поля и его продолжение на границе структуры.