Computational modelling of flow and pollution transportation in open channel with complicated geometry and bottom relief

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ABSTRACT: A 2-D depth-average computational model based on the shallow water theory including aspects of non-uniform bottom relief has been developed. An explicit differential scheme formulated on the "one-sign" transport matrixes has been used for solving the boundary problem of non-conservative from. Corrections to the differential scheme are proposed to provide good agreement with available basic solutions of the equations system. In couple with the flow module, the diffusive-transport equation has also been solved respectively at each time step to predict the depth-average concentration or surface concentration of pollution spreading in a channel that contained islets.

1 INTRODUCTION

Predicting of water flow regime in an open channel such as river branch and estuary is necessary not only for civil engineering projects where many man-made constructions to be introduced but also be importance for controlling water environment as many pollution sources may discharge out form industries or catastrophes. Recently many 2-D mathematical models have been proposed basing on the shallow water theory instead of an expensive full 3-D model. Among those models, some are taken in to account various phenomenon that could influence on the flow characteristics such as bottom friction, bed evaluation due to sediment transport, as well as secondary flow in curved channels, see Minh Duc (1996). In this paper a simple approach for solving numerically the depth-average shallow water equations are introduced taking into account the sense of disturbance transportation during integrating the governing equations. This technique is especially advanced when the flow is rapid or the flow direction may change with time, for example in an estuary where the flow direction usually changes periodically due to tidal influence. By coupling the above-mentioned hydrodynamic module with solving the diffusion equation time-to-time, a transport-diffusion model is constructed which provides a useful instrument to predict the depth-average concentration of pollution along the channel once there are polluted sources spread out. Computational experiments have been carried out for a river branches in Mekong delta of Vietnam with real shape and bottom topography.

2 CALCULATION METHOD

2.1 *Governing equations*

Considering that the fluid is incompressible, homogeneous and viscous, the pressure distribution is quasi-hydrostatic, in a Cartesian coordinate system x, y, z the three-dimensional problem described by the Reynolds' equations is reduced by depth-averaging to a two-dimensional problem knew as "shallow water theory". The equation is expressed in mean values of the velocity components u and v and the water depth h and applicable for a shallow domain in which the vertical characterized dimension is rather small in comparisons with horizontal one. For an open channel, the Coriolis force that is due to the earth's rotation around its axis is negligible therefore the equation system is as follows, see Abbot (1983):

$$\frac{\partial h}{\partial t}+\frac{\partial uh}{\partial x}+\frac{\partial vh}{\partial y}=0 \quad (1)$$

$$\frac{\partial uh}{\partial t}+\frac{\partial uuh}{\partial x}+\frac{\partial uvh}{\partial y}+gh\frac{\partial h}{\partial x}=gh\frac{\partial H}{\partial x}+h\tau_x \quad (2)$$

$$\frac{\partial vh}{\partial t}+\frac{\partial uvh}{\partial x}+\frac{\partial vvh}{\partial y}+gh\frac{\partial h}{\partial y}=gh\frac{\partial H}{\partial y}+h\tau_y \quad (3)$$

where H =water depth at still state; g =gravitation acceleration; τ_x, τ_y = shear stress caused by bottom friction and viscous between the fluid layers:

$$\tau_x=-c_f\sqrt{u^2+v^2}hu+\nu h\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \quad (4)$$

$$\tau_y=-c_f\sqrt{u^2+v^2}hv+\nu h\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \quad (5)$$

where c_f = bottom friction coefficient which is proposed by various experts in term of empirical formulas; ν = dynamical viscosity. One of such well know empirical formulas for bottom friction coefficient that has been used in this programming is the Chezy formula, see Minh Duc (1996):

$$c_f=\frac{g}{c_z^2} \quad (7)$$

$$c_z=18\log\left(\frac{h}{k_s}\right) \quad (8)$$

where k_s = hydraulic radius of the channel is a further empirical parameter

When in flow appears pollution substance, it spreads along the channel by mainly two processes: diffusion and transportation. A part of substance may be neutralized by some reasons. Mathematically, these processes can be described by the following equation, see Zgurovsky et al. (1997):

$$\frac{\partial\varphi}{\partial t}+u\frac{\partial\varphi}{\partial x}+v\frac{\partial\varphi}{\partial y}+\sigma\varphi=\frac{1}{h}\frac{\partial}{\partial x}\left(\mu h\frac{\partial\varphi}{\partial x}\right)+\frac{1}{h}\frac{\partial}{\partial y}\left(\mu h\frac{\partial\varphi}{\partial y}\right)+\frac{1}{h}q_i\delta(x_i,y_i) \quad (9)$$

where φ = depth-average concentration of pollution, σ = coefficient characterized neutralization rate of pollution, μ =diffusive coefficient and q_i =discharge rate of pollution at point (x_i, y_i). Function δ is taken unit value at point (x_i, y_i) but turns to zero at others else.

2.2 *Boundary conditions*

In order to solve the differential equations (1)-(3), boundary conditions must be specified for all independent variables. At the upstream boundary, the distribution of velocity is set proportional to the flow depth as in flow condition. At the outlet boundary, the flow depth is prescribed and the stream-wise gradient of other variables are set to zero, implying fully developed flow. In addition the velocity is corrected by considering that the total inflow to the computational do main must equal the outflow from the domain since the channel bottom as well as solid wall restrained flow within the channel are assumed impermeable. At the solid boundary, for the viscous fluid the sleeping condition was used i.e. both flow components were set to zero $u = 0, v = 0$ while the gradient of water elevation $\varsigma = h - H$ is also set to zero. For diffusive-transport equation, the gradient of concentration $\partial\varphi/\partial n$ is set to zero at the solid wall as well as at outlet boundaries. Discharge of pollution q is given at several points in the flow domain at prescribed periods of time.

2.3 *Numerical method*

The explicit finite differential method, so called the "against flow" scheme is used to solve the above mentioned boundary problem, see Samarsky (1983), Beliaev. & Khrutch (1984). For this, equations (1)-(3) to be written in non-conservative vector form as follows:

$$\frac{\partial U}{\partial t} + A\frac{\partial U}{\partial x} + B\frac{\partial U}{\partial y} = F \tag{10}$$

where U = column vector of h, u, v; F = column vector of the right parts of equations (1)-(3); A, B = transformation matrixes:

$$A = \begin{pmatrix} u & h & 0 \\ g & u & 0 \\ 0 & 0 & u \end{pmatrix};\ B = \begin{pmatrix} v & 0 & h \\ g & v & 0 \\ 0 & 0 & v \end{pmatrix} \tag{11}$$

In order to apply the "against flow" method, the matrixes A, B are then broken up as sum of sign-unchanged matrixes:

$$A = A^{+} + A^{-};\ B = B^{+} + B^{-} \tag{12}$$

so that, the determinants of the matrixes with plus-sign are not negative while the determinants of the minus - sign ones are not positive.

Equation (10) after integrating by using explicit finite differential operators is now taken following form:

$$U^{n+1} = U^{n} - \Delta t\left(A^{+}\Lambda_{x}^{+}U^{n} + A^{-}\Lambda_{x}^{-}U^{n} + B^{+}\Lambda_{y}^{+}U^{n} + B^{-}\Lambda_{y}^{-}U^{n} + F^{n}\right) \tag{13}$$

where $\Lambda_{x}^{\pm}, \Lambda_{y}^{\pm}$ = discrete differential operators:

$$\Lambda_{x}^{+}U = \frac{U_{i,j} - U_{i-1,j}}{\Delta x};\ \Lambda_{x}^{-}U = \frac{U_{i+1,j} - U_{i,j}}{\Delta x} \tag{14}$$

$$\Lambda_{y}^{+}U = \frac{U_{i,j} - U_{i,j-1}}{\Delta y};\ \Lambda_{y}^{+}U = \frac{U_{i,j+1} - U_{i,j}}{\Delta y} \tag{15}$$

The condition for stability of the scheme is knew as Courant-Levis condition:

$$\Delta t \le \frac{\min\{\Delta x, \Delta y\}}{\max\{|u|, |v|, a\}} \tag{16}$$

where $a = \sqrt{gh}$ is also called "long wave transportation speed".

Obviously, it is required that the numerical solutions of equation (10) should provide fully agreement with its basic analytical solutions. In this case, we note that with any bottom profile $H = H(x, y)$, column vector $U=(H(x,y),0,0)$ yet to be a basic solution of (10). There for a correction should be applied to the discrete integration to provide this agreement.

For solving diffusive-transport equation (9), assuming that the flow field is predetermined and the following differential scheme has been used:

$$u\frac{\partial\varphi}{\partial x}=\frac{u+|u|}{2}\Lambda_x^+\varphi+\frac{u-|u|}{2}\Lambda_x^-\varphi;\quad v\frac{\partial\varphi}{\partial y}=\frac{v+|v|}{2}\Lambda_y^+\varphi+\frac{v-|v|}{2}\Lambda_y^-\varphi \qquad (17)$$

Thus instead of solving equation system (1)-(3) and (9) simultaneously, at every time step, firstly the velocity components are calculated via (10) then these flow parameters are used for calculating pollution concentration by solving equation (9) using differential scheme (17) in which, the space differential terms are taken forward or backward depends on the flow component in that direction is positive or negative respectively.

3 COMPUTATIONAL RESULTS

As a computational experiment, the model is applied for a river branch in Mekong delta in Vietnam. The shape of the river branch is formed with and an islet as showed in figure 1. The length of the computational domain is about 2180m, while the wide of river branch is changing from upstream to its outlet boundary with the minimum and maximum wide are of about 300m and 1200m respectively. The river branch contains an islet at the middle that divides the flow domain into two parts. Computational domain forms a regular orthogonal mesh of 82x110 grid points, where space step is of 20m. Maximum water depth is somewhat of 12m while the shallowest point is of 2m water depths along solid boundaries.

3.1 *Steady case*

At upstream boundary the water discharge is given at rate Q=1000m^3/s. Taking initial condition as water in the domain remains still at moment t=0, the flow in generally will reach fully steady state after 5000s. The maximum time step is recommended 0.75s while the model is unstable thus after hundreds seconds with the time step 1s. The distribution of velocity magnitude at full steady state is shown in figure 2. From the figure may see that the velocity magnitude at straight channel is much higher than those in curved one. In figure 3 illustrated the development of pollution concentration along the river when a pollution source of q=100kg/s is discharged out at point B coordinate (x=600m,y=1860m). The pollution is spreading along the riverbank. The highest concentration at down stream is about 0.5 kg/m^3 while the value near discharging point is a bout 1.5kg/m^3. The contour lines are curved along the riverbank. In figure 4 shown concentration contours after 30000s of discharging of pollution at point A at upstream (x=220m, y=2180m). The existence of the islet is also obviously seen on the contours picture as the contour lines are fitted with the shape of the flow domain.

3.2 *Unsteady case*

To carry out an experiment for unsteady case, the water level at down stream is proposed to be fluctuated periodically by time $\varsigma = d\sin(2\pi t/T)$, where d=1m and T=3600s. Pollution concentration in this case is considered as the salinity at sea. Flow discharge at upstream is given constantly Q=700cm^3/s.

The concentration of salt is given constantly at outlet boundary φ_0=1kg/m^3. Figures 5-6 show the salinity contours lines at different time in a period. The salinity in the straight channel is much higher in the curved one. Figure 7 illustrates water level as a function of time at point C (x=1580m, y=1180m). The water level also so fluctuates with the same rule as given at the outlet boundary. Slight difference in phase and amplitude between the outlet boundary and inside the flow domain may be explained as the influence of topography as well as bottom friction. Relative salinity c=φ/φ0 at point C is varied periodically by time after 20 hours, when the flow is fully developed and is shown in figure 8. Figure 9 shows also relative salinity displacement but for point D (x=640m, y=1460m) near the islet. The difference of the curved shape of salinity variation between the two points proves that the existence of an islet within the flow domain leads to a more complicated picture of salinity distribution.

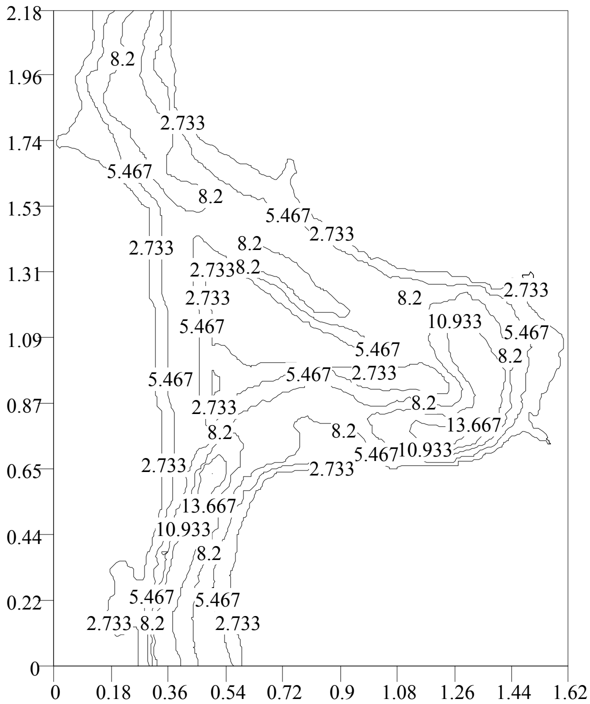

Figure 1. Water depth profile and topography

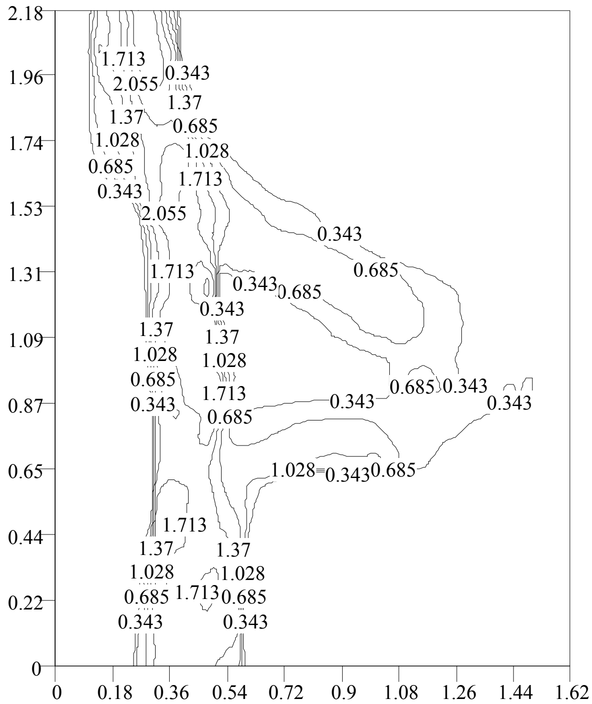

Figure 2. Velocity magnitude distribution, steady case, Q=1000

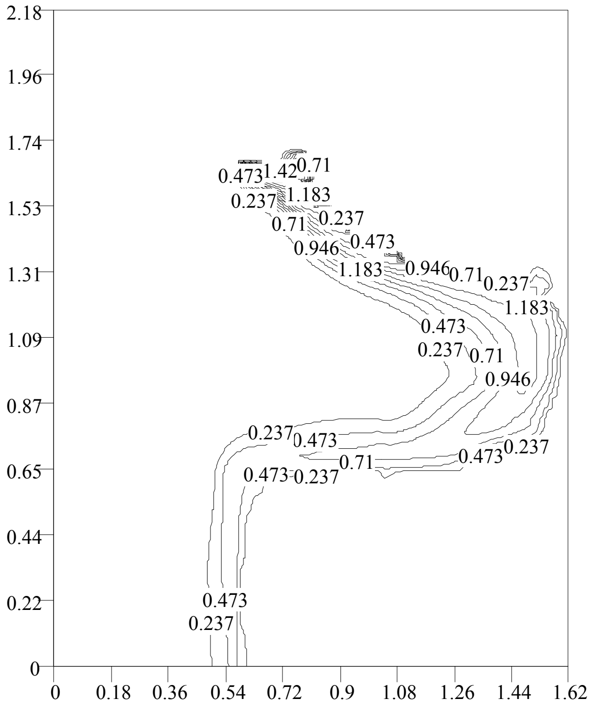

Figure 3. Concentration contours in steady flow after 30000s of discharging at point A

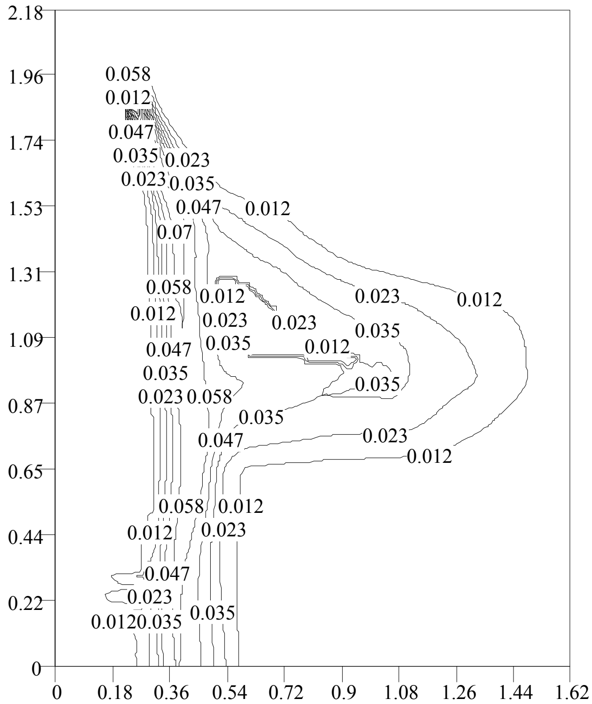

Figure 4. Concentration contours in steady flow after 30000s of discharging at point B

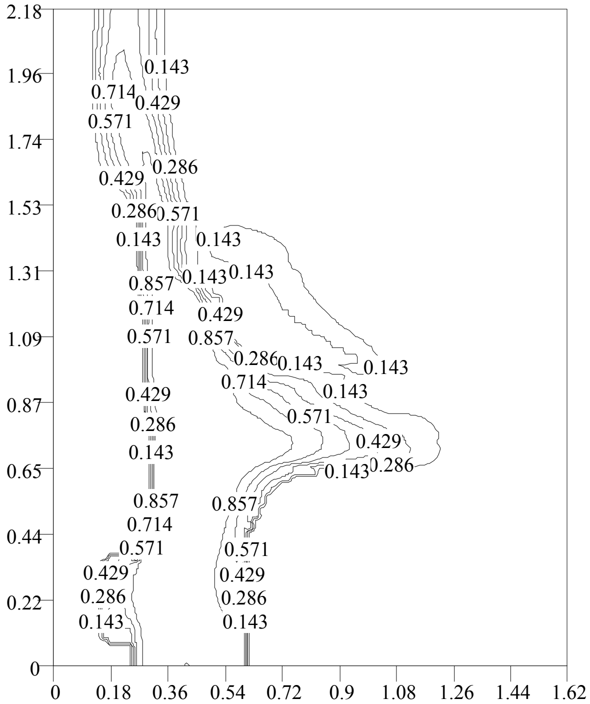

Figure 5. Salinity concentration after 27000s, T=3600s, Q=700, a=1m

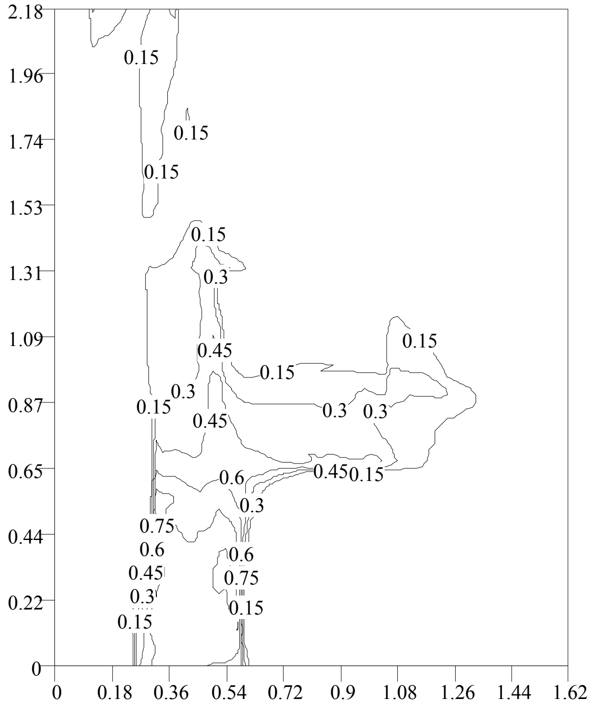

Figure 6. Velocity magnitude distribution 28000s, T=3600s, Q=700, a=1m

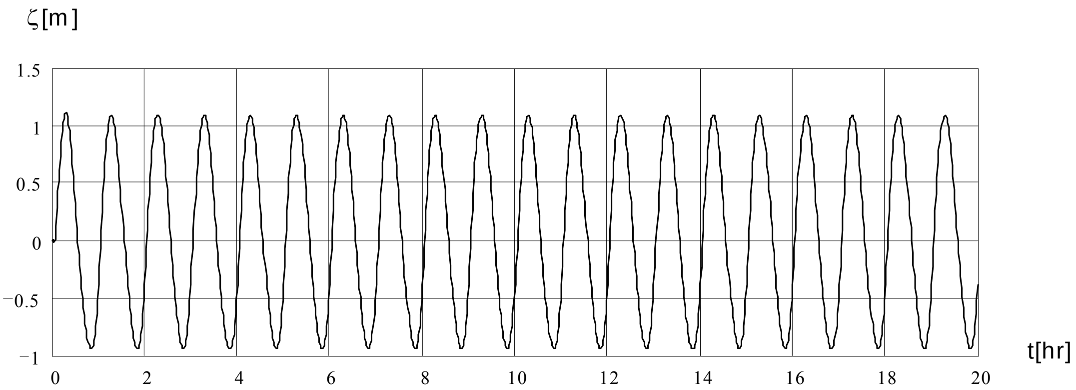

Figure 7. Water level variation at point C

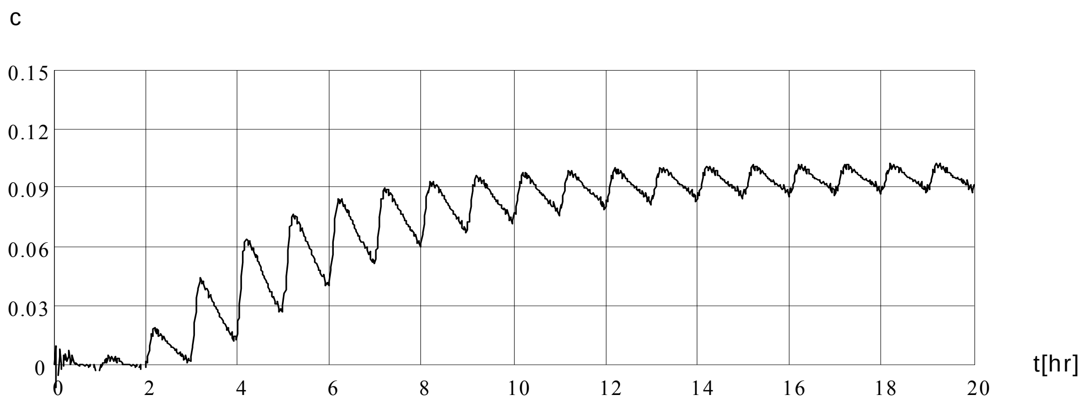

Figure 8. Salinity variation at point C

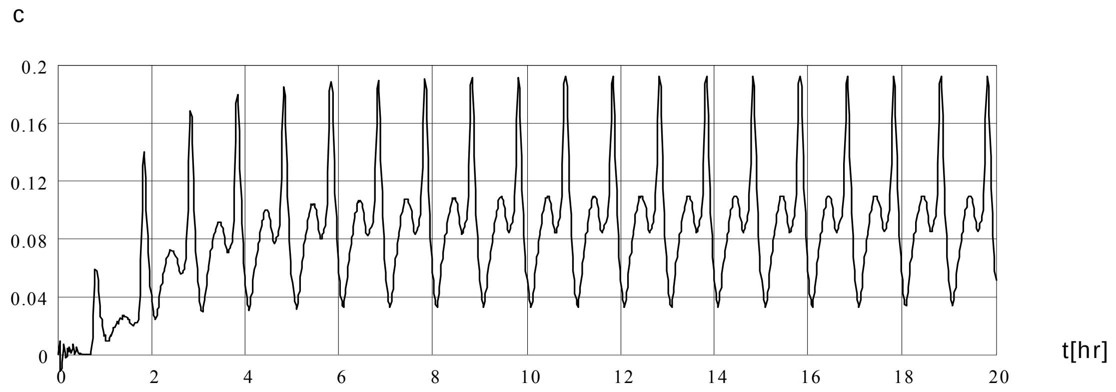

Figure 9. Salinity variation at point D

4 CONCLUSION

The proposal model provides a simple instrument for calculating hydrodynamic regime as well as pollution transportation in an open channel. The computational results are reasonable in term of quantity. To verify the model qualitatively requires more field-measuring data. The model is applicable for a channel with complicated topography by using markers to identify the dry points and wet points in the computational domain. From environment point of view, the model is applicable to define authorized limited discharge rate of a polluted substance up stream that should not be harmful to down stream environment.

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Duongs N.H., Kručs V.K. Plūsmas un piesārņojuma transporta skaitliskā modelēšana atklātā kanālā ar sarežģītu ģeometriju un apakšas virsmu. *Divdimensionāls skaitliskais modelis, kurš izmanto seklā ūdens teoriju, ievērojot neplakanas kanālā dibena virmas ietekmi. Tiešā diferenciāla shēma ir formulēta, izmantojot transporta matricu. Shēma izmantota robežproblēmas risināšanai nekonservatīvā formā. Tiek piedāvāti tādi diferenciālas shēmas uzlabojumi, kuri nodrošina labu attīstību vienādojumu sistēmas fundamentālajiem risinājumiem. Izmantojot plūsmas modeli, katram laika solim atrisināts difūzīvā transporta vienādojums. Risinājums dod vidējo piesārņojuma koncentrāciju dziļumam un arī virsmas koncentrācijas kanālā, kurā ir salas.*

Дуонг Н.Х., Круч В.К. Численное моделирование потока и траспорта загрязнения в открытых каналах, имеющих сложную геометрию и поверхность дна. *Двухмерная численная модель основана на теории неглубокой воды с учетом неплоского дна. Явная дифференциальная схема, сформированная на базе матриц транспорта, была использована для решения краевой задачи в неконсервативной форме. Предложены коррекции этой схемы, которые обеспечивают хорошее соответствие с фундаментальными решениями для системы уравнений. В сочетании с модулем потока, уравнение диффузного траспорта было решено для каждого временного шага. Полученное решение дает среднюю концентрацию загрязнения по глубине, а также поверхностную концентрацию, которая распространяется в канале содержащем острова.*