A nonlinear lightguide as a transmission medium of ultrashort chirped hyperbolic secant shape and Gaussian pulses

T. Kaczmarek
Kielce University of Technology, Institute of Telecommunications and Photonics, Kielce, Poland
Warsaw University of Technology, Institute of Electronic Systems, Warszawa, Poland

C. Kaczmarek
Warsaw University of Technology, Institute of Electronic Systems, Warszawa, Poland

Keywords: soliton, Nonlinear Schrödinger Equation, chirp

ABSTRACT: The propagation of ultrashort chirped hyperbolic secant shape and Gaussian pulses in a nonlinear lossless fiber is considered. Nonlinear Schrödinger Equation is solved numerically for complex initial conditions using the Split-Step Fourier and the Beam Propagation Methods. It is demonstrated that a hyperbolic secant shape pulse is less sensitive to destructive influence of the initial chirp than a Gaussian pulse.

1 INTRODUTION

The fundamental soliton is such an optical pulse, which can propagate without change in shape for arbitrarily long distances in a lossless fiber (Majewski 1993, Zakharov & Shabat 1972). It is due to the mutual compensation of the dispersion and the nonlinearity of the fiber. Generation of the fundamental soliton is possible if and only if the peak power of the pulse exceeds a certain threshold value. In an ideal case the initial pulse should have hyperbolic secant shape and should be free from chirp. The satisfying of the condition of the shape is difficult and generation of the pulse, which is completely free from chirp is impossible in practice. Therefore the investigation of the influence of the shape and chirp on the pulse generation appears to be useful.

In the earlier paper (Kaczmarek 1999) influence of the initial chirp on the hyperbolic secant shape pulse was investigated. The Nonlinear Schrödinger Equation (NLSE)

$$j\frac{\partial A}{\partial z}+\frac{1}{2}\beta_2\frac{\partial^2 A}{\partial T^2}+\gamma|A|^2 A=0\,, \tag{1}$$

where A = envelope function, γ = nonlinearity, β_2 = dispersion, z = spatial coordinates, and T = time coordinates, was solved using the Split-Step Fourier Method (SSFM) for the following complex initial condition

$$A(z=0,T)=N\sec h\left(\frac{T}{T_0}\right)\exp\left(-j\frac{C}{2}\frac{T^2}{T_0^2}\right), \tag{2}$$

where N = order of the soliton, T_0 = initial pulse width, and C = chirp parameter. Similarly the standard form of the NLSE

$$j\frac{\partial q}{\partial \zeta}+\frac{1}{2}\frac{\partial^2 q}{\partial \tau^2}+|q|^2 q=0\,, \tag{3}$$

where q = normalised envelope function, ζ = normalised spatial coordinates, and τ = normalised time coordinates, was solved using the Beam Propagation Method (BPM) for the complex initial condition similar to the condition mentioned above (2).

In an another paper (Kaczmarek 2000) the method of the designation of the critical value of the chirp parameter was proposed. The chirp parameter C reaches the critical value C_{kr}, when the value of the stationary peak amplitude $\left|A\right|_{\max,stationary}$ of the fundamental soliton is equal to zero.

In the present paper it is intended to determine C_{kr} for the Gaussian initial pulse with the use of the SSFM and BPM. In the case of the hyperbolic secant shape initial pulse it is going to be proven, that the application of the third stage of the evaluating of C_{kr}, i.e. extrapolation, is not possible because of the shape of the function $\left|A\right|_{\max,stationary} = f(C)$. Finally it is proved, that a hyperbolic secant shape pulse is less sensitive to the destructive influence of the chirp than a Gaussian pulse.

2 METHODS

The evaluation of C_{kr} can be divided into three stages. At the first stage the temporary envelope variability of the pulse with respect to the propagation distance should be calculated. For this purpose the NLSE should be solved numerically using the BPM and the SSFM. The use of the numerical methods is recommended, because the analytical solution of the NLSE (1) for complex initial condition (2) is difficult to obtain.

The SSFM and the BPM have physical grounds. The idea is based on the separate consideration of the consequences of the nonlinearity and the dispersion in a short segment of the guide. In case of SSFM it can be represented schematically, when (1) is expressed in the operator form (Hasegawa & Tappert 1972, Agrawal 1989)

$$\frac{\partial A}{\partial z} = (\boldsymbol{D} + \boldsymbol{N})A, \tag{4}$$

where $\boldsymbol{D} = -j\frac{1}{2}\beta_2\frac{\partial^2}{\partial T^2}$ = dispersion operator, and $\boldsymbol{N} = j\gamma\left|A\right|^2$ = nonlinearity operator. After some transformation, the optical field can be expressed as follows:

$$A(z+h,T) = F^{-1}\left\{\exp(h\boldsymbol{D})F\left[\exp(h\boldsymbol{N})A(z,T)\right]\right\} + R\left(h^2\right), \tag{5}$$

where F denotes the Fourier transformation and h is the length of the step. In case of BPM the pulse shape after travelling a distance $\Delta\zeta$ is expressed as a function of the input pulse $q(\tau,\zeta)$ in the following way (Yevich & Hermanssen 1983)

$$q(\tau,\zeta+\Delta\zeta) = \boldsymbol{GH}\left[\boldsymbol{G}q(\tau,\zeta)\right]\boldsymbol{G}q(\tau,\zeta) + O\left(\Delta\zeta^3\right), \tag{6}$$

where $\boldsymbol{G} = \exp\left(j\frac{1}{4}\Delta\zeta\frac{\partial^2}{\partial\tau^2}\right)$ = dispersion operator, $\boldsymbol{H} = \exp\left(j\Delta\zeta\left|q\right|^2\right)$ = nonlinearity operator. Mathematically, the equation (6) is evaluated by consecutively Fourier transforming the pulse envelope q before multiplying it by the operator $\boldsymbol{G}$ and inverse transforming the result before applying the operator $\boldsymbol{H}$.

At the second stage, in order to calculate the stationary peak amplitude value $\left|A\right|_{\max,stationary}$ of the envelope, an averaging filter with a variable window is used. In case of taking advantage of the SSFM at the first stage, at the second one the peak amplitude as a function of the distance $\left|A\right|_{\max} = f(z)$ should be filtered. $\left|A\right|_{\max} = f(z)$ is such a function, that independently of the chirp parameter value C, consist of 101 samples. The averaging filter functions in agreement with the following algorithm (Hagel & Zakrzewski 1984)

$$f_j = \frac{\sum_{i=j-n}^{j}\left|A\right|_{\max}(z_i)}{n}, \tag{7}$$

where f_j is j - th sample of the waveform $|A|_{\max} = f(z)$ after the filtering, n is the variable length of the window of the filter. If the data obtained as the result of the BPM application has to be filtered, in the equation (7) in place of $|A|_{\max}$, $|q|_{\max}$ should be inserted.

At the third stage, which in this paper is used only in case of the Gaussian initial pulse, the critical value C_{kr} of the chirp parameter C is estimated. In this end, a well-known rational extrapolation algorithm (Press & Vatterling & Teukolsky & Flannery 1992) is used. The estimation of the critical value of the chirp parameter consists in seeking a zero of the curve $|A|_{\max,stationary} = f(C)$ ($|q|_{\max,stationary} = f(C)$ in case of the BPM).

3 RESULTS

Making use of the formulas (5) and (6), computations were carried out for the hyperbolic secant shape and the Gaussian pulse when $\beta_2 = -1.6\dfrac{ps^2}{km}$, $\gamma = 1.6W^{-1}km^{-1}$ for selected values of the chirp parameter C . The dispersion and the nonlinearity parameters were chosen so, that the effective fiber core section was equal to $A_{eff} = 80\mu m^2$. The acceptance of the step $\Delta\zeta = 0.0256$ in equation (6) had in view the comparison of the results received with the aid of the SSFM and the BPM.

At the third stage of the evaluation of the C_{kr} , the following results were obtained (in case of the initial pulse being rectangular):

$C_{kr+} = 1{,}841227$ (BPM); $C_{kr+} = 1{,}831733$ (SSFM); the arithmetic average $C_{kr+} = 1{,}83648$;

$C_{kr-} = -1{,}817177$ (BPM); $C_{kr-} = -1{,}82425$ (SSFM); the arithmetic average $C_{kr-} = -1{,}8207135$.

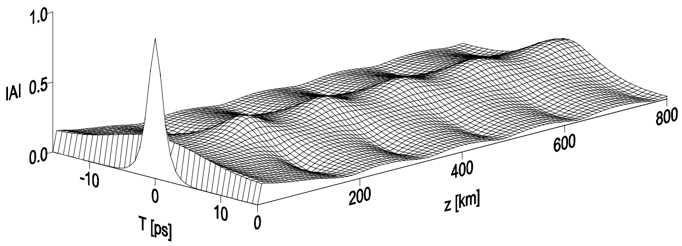

Figure 1.a. Evolution of the hyperbolic secant shape pulse when $C = -1.375$

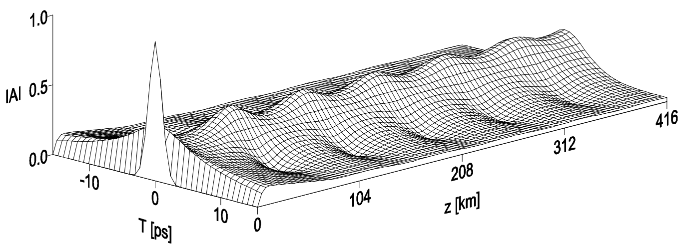

Figure 1.b. Evolution of the Gaussian pulse when $C = -1.375$

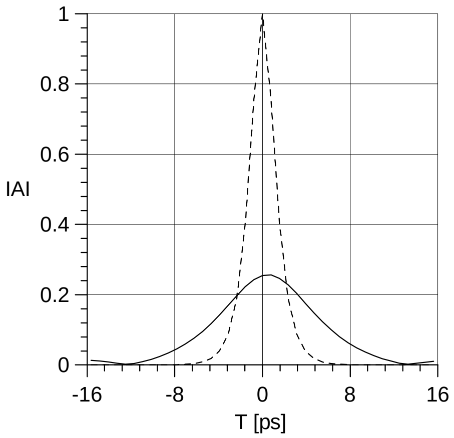

Figure 2.a. Evolution of the hyperbolic secant shape pulse when $C = -1.375$; dashed line – initial shape, solid line - shape of the pulse after travelling a distance of $z = 776km$

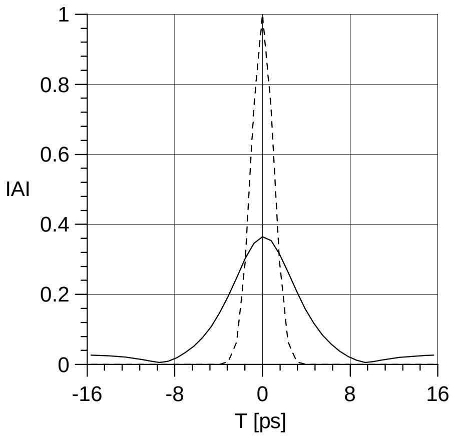

Figure 2.b. Evolution of the Gaussian pulse when $C = -1.375$; dashed line – initial shape; solid line - shape of the pulse after travelling a distance of $z = 361.92km$

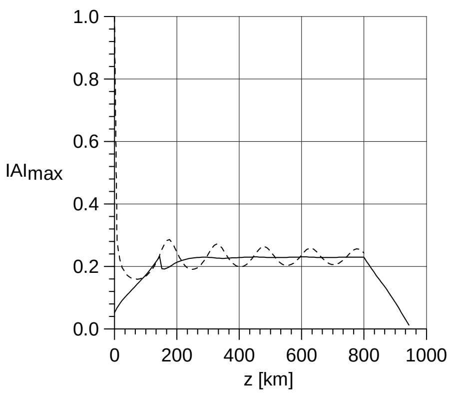

Figure 3.a. Graphical interpretation of the functioning of the averaging filter; dashed line – evolution of the peak amplitude of the hyperbolic secant shape pulse (i.e. the shape of the curve $|A|_{\max} = f(z)$) when $C = -1.375$; solid line – current average of the curve $|A|_{\max} = f(z)$

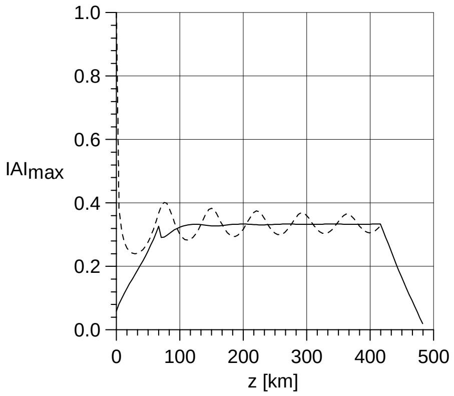

Figure 3.b. Graphical interpretation of the functioning of the averaging filter; dashed line – evolution of the peak amplitude of the Gaussian pulse (i.e. the shape of the curve $|A|_{\max} = f(z)$) when $C = -1.375$; solid line – current average of the curve $|A|_{\max} = f(z)$

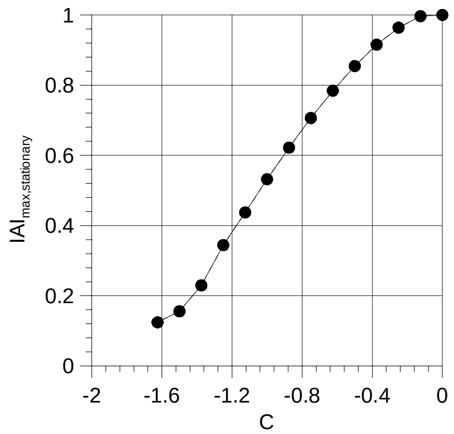

Figure 4.a. Rational extrapolation is not suitable for determining the zero point (C_{kr-}) of the curve $|A|_{\max,stationary} = f(C)$ in case of hyperbolic secant shape pulse because of the asymptotic character of the zeroing due to inflexion point occurrence

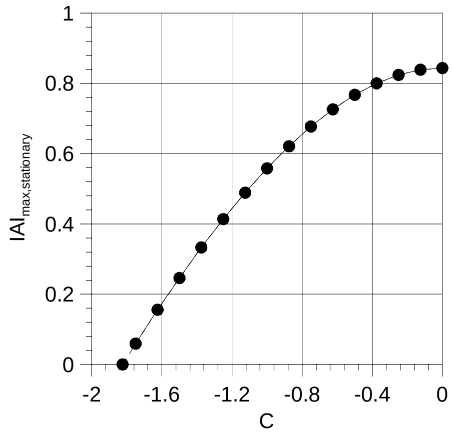

Figure 4.b. Graphical interpretation of the rational extrapolation C_{kr-} in case of the Gaussian pulse, i.e. zero point search of the curve $|A|_{\max,stationary} = f(C)$

4 DISCUSSION

Figures 1.a and 1.b show the shape variation of the pulse as a function of the propagation distance for the value of the chirp parameter $C = -1.375$. Figure 1.a concerns the case of the hyperbolic secant shape initial pulse. Figure 1.b shows the transformation of the Gaussian pulse into the soliton. The maximum influence of the nonlinearity can be seen at such points where the peak amplitude of the pulse reaches the local maximum (i.e. the local narrowing of the pulse). On the contrary the cumulating of the influence of the dispersion can be observed where the peak amplitude of the pulse reaches the local minimum (i.e. the local broadening of the pulse). The last narrowing of the hyperbolic secant shape pulse from figure 1.a is presented in figure 2.a (solid line). The analogous narrowing of the Gaussian pulse can be seen in figure 2.b. Variability of the peak amplitude value of the pulse $|A|_{\max}$ in case of the hyperbolic secant shape pulse is presented as the dashed line in figure 3. a. The solid line (fig. 3.a) illustrates the graphical interpretation of the functioning of the averaging filter, the use of which enables the determining of the stationary peak amplitude value $|A|_{\max,stationary}$ of the pulse for the adequate value of the chirp parameter C. Figure 3.b is corresponding to figure. 3.a except for the fact, that figure 3.b. refers to the Gaussian pulse. Making a comparison between figure 1.a and 1. b, between 2.a and 2.b as well as 3.a and 3.b it can be noticed, that for the value of the chirp parameter $C = -1.375$ the stationary peak amplitude value $|A|_{\max .stationary}$ of the pulse, approaches zero faster for the hyperbolic secant shape than for the Gaussian pulse. The comparison between figure 4.a and 4.b confirm this fact but only for $|C| < 1.375$. For $|C| > 1.375$, in case of the hyperbolic secant shape $|A|_{\max,stationary}$ approaches zero slower (asymptotically) than in case of the Gaussian pulse. It is due to the occurrence of the flexion point on the curve $|A|_{\max,stationary} = f(C)$ in case of the hyperbolic secant shape pulse.

5 CONCLUSION

On the basis of the performed calculation it is found that a hyperbolic secant shape pulse is less sensitive to the destructive influence of the chirp than a Gaussian pulse. Furthermore in case of the Gaussian pulse the absolute values of the critical chirp parameter satisfy the following condition $|C_{kr+}| > |C_{kr-}|$. From the above condition arises the fact that in case of the Gaussian pulse a positive chirp is less dangerous than a negative one.

REFERENCES

Agrawal, G.P. 1995. *Nonlinear Fiber Optics. Second Edition*. San Diego: Academic Press.

Hagel, R. & Zakrzewski, J. 1984. *Miernictwo dynamiczne*. Warszawa: Wydawnictwa Naukowo Techniczne.

Hasegawa, A. & Tappert, F. 1973. Transmission of stationary nonlinear optical pulses in dispersive dielectric fibers I. Anomalus dispersion, *Appl Phys. Lett.* 23(3): 142-144.

Kaczmarek, T. 1999. Propagation of chirped ultra-short optical pulses. *International Conference on Transparent Optical Networks ICTON'99. Conference Proceedings. Kielce 9-11 June 1999*. Warszawa: Instytut Łączności.

Kaczmarek, T. 2000. Propagation of chirped rectangular pulses in nonlinear lessless optical wave guide, *International Conference on Transparent Optical Networks ICTON 2000. Conference Proceedings.* Gdańsk *5-8 June 2000*. Warszawa: Instytut Łączności.

Majewski, A. 1993. Nieliniowa optyka światłowodowa. Zagadnienia wybrane. Warszawa: Wydawnictwa Politechniki Warszawskiej.

Press, W.H. & Vatterling, W.T. & Teukolsky, S.A. & Flannery, B.P. 1992. *Numerical Recipes in Fortran 77. Second Edition.* Cambridge: Cambridge University Press.

Yevich, D. & Hermanssen, B. 1983. Soliton analysis with the Propagating Beam Method. *Optics Comm*. 47(2):101-106.

Захаров В.Е & Шабат А.Б. 1971. Точная теория двумерной самофокусировки и одномерной автомодуляции волн в нелинейных средах. *Журнал экспериментальной и теоретической физики*. Т. 61. Вып. 1(7): 118-34.

Tomasz Kaczmarek, *Mgr inz.*
Kielce University of Technology, Institute of Telecommunications and Photonics, Kielce, Poland
Warsaw University of Technology, Institute of Electronic Systems, Warszawa, Poland
Address: Al. Tysiąclecia P.P. 7, 25-314 Kielce, Poland
Phone: (+4841) 3424229; e-mail: tkaczmar@tu.kielce.pl

Cezary Kaczmarek, *Mgr inz.*
Warsaw University of Technology, Institute of Electronic Systems, Warszawa, Poland

Kačmareks T., Kačmareks C. Nelineārs gaismas vads kā vide ultraīsu impulsu pārraidei, kuriem ir hiperboliskā sekante un Gausa līknes forma ar traucējumiem.

Apskatīta ultraīsu impulsu pārraide nelineārā gaismas vadā bez zudumiem. Impulsiem ir hiperboliskā sekante un Gausa līknes forma ar traucējumiem. Tiek skaitliski atrisināts nelineārais Šrēdingera vienādojums kompleksiem sākuma noteikumiem, izmantojot dalītā soļa Furjē un stara izplatīšanas metodes. Tiek pierādīts, ka hiperboliskās sektantes formas impulsi nav tik jūtīgi pret kaitīgo sākuma traucējumu ietekmi kā Gausa tipa impulsi.

Качмарек Т., Качмарек Ц. Нелинейный волновод как среда для передачи ультра-коротких импульсов, которые имеют форму гиперболического секанта и кривой Гаусса с помехами.

Рассмотрена передача ультра-коротких импульсов в нелинейном световоде без помех. Импульсы имеют форму гиперболического секанта и кривой Гаусса с помехами. Получено численное решение нелинейного уравнения Шредингера применяя методы Фурье с дробным шагом и распространения пучка. Доказано, что импульсы имеющие форму гиперболического секанта менее чувствительны к разрушающему действию начальных помех, чем импульсы типа кривой Гаусса.