Waveform reconstruction of impact force using a frequency domain technique and one-point strain measurement

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ABSTRACT: In the paper a transformational impact force reconstruction method is presented, which features a one-point strain measurement in a mechanical transducer. To determine the spectral transmittance of a section of the mechanical transducer necessary in the reconstruction process, strain waves propagating in the transducer are used: the incident, and the first reflected one, created by the impact. Restrictions concerning the impact duration as well as the strain gauge localization on the mechanical transducer are given, which assure the non interference of the above mentioned waves in the strain gauges. The reconstruction results of impact force waveforms generated during impacts of steel spheres in the front face of a steel bar are presented. The consistency of the obtained reconstruction results with the Hertz Impact Theory with respect to the duration and amplitude of the impact force was evaluated.

1 INTRODUCTION

Extending the application of the Hopkinson bar as a mechanical transducer in measurements of impulsive mechanical quantities such as force and pressure, as well as their derivatives, onto the dispersive operating range requires the correction of the dispersion's influence on processing (Kaczmarek 2001). There exist analytical and experimental methods of correcting dispersive distortion in the Hopkinson bar. In the analytical methods the bar's dispersive characteristics are used, obtained as a result of numerically solving the equation of movement of the bar, with the input being a harmonic function. On the ground of these characteristics appropriate phase shifts are introduced to particular frequency components, thus correcting dispersive distortions(Follansbee & Franz 1983, Gorham 1983, Gong et al. 1990). The methods mentioned above produce satisfying results for small distortions.

In the experimental methods the dispersive characteristic of bar used for measurements is determined empirically. For this purpose the one-point measurement as well as the two-point measurement of the strain inside the bar is applied. The one-point measurement of the strain inside the bar created as a result of a low velocity impact of a small sphere into its front face is the basis of evaluating the phase characteristic of the bar in the paper (Gorham & Wu 1996). The output of a sensor, in which the two-point strain measurement is used, enables not only the determining of the dispersive characteristic of the bar but also the realization of the input waveform reconstruction (Kaczmarek & Drobnica 1998, Kaczmarek 2000).

In the presented paper a means of reconstructing impact force waveforms with the use of the one-point measurement of the strain inside the bar is demonstrated . The reconstruction process was carried out in the frequency domain using the Fourier Transform and a regularization filter of the form given in the work (Kaczmarek 2001). For calculation of the spectral transmittance of a section of the bar, required in the reconstruction process, strain waves are used, generated as a result of the impact: the incident and the first reflected one. Therefore the output signal of such a one-point strain sensor contains information analogical to that of a two-point sensor output. The one-point sensor co-operates with a single channel conditioning and signal processing circuit, which allows the required processing accuracy to be achieved much more easily than it is the case when applying the two-point sensor with a two channel conditioning and signal processing circuit.

2 THE IMPACT FORCE SENSOR WITH ONE-POINT STRAIN MEASURING

The essence of the one-point strain processing sensor's operation is best described using the one dimensional equation of the bar's motion, witch has the form:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c_0^2}\frac{\partial^2 u}{\partial t^2} \qquad (1)$$

where u = is the displacement; $c_0 = (E/\rho)^{0,5}$ = the velocity of longitudinal stress waves propagation inside the bar; E = Young's modulus; ρ = bar's material density.

The solution of equation (1) includes two functions, that represent waveforms propagating in the positive and the negative directions of the x-axis, and are noted:

$$u(x,t) = u_1\left(t - \frac{x}{c_0}\right) + u_2\left(t + \frac{x}{c_0}\right) \qquad (2)$$

The longitudinal strain waves can be written in the form:

$$\varepsilon(x,t) = \varepsilon_1\left(t - \frac{x}{c_0}\right) + \varepsilon_2\left(t + \frac{x}{c_0}\right) \qquad (3)$$

where $\varepsilon(x,t) = \partial u(x,t)/\partial x$.

Other mechanical quantities, such as the particle velocity v, stress σ, can be determined from relations (2) and. (3).

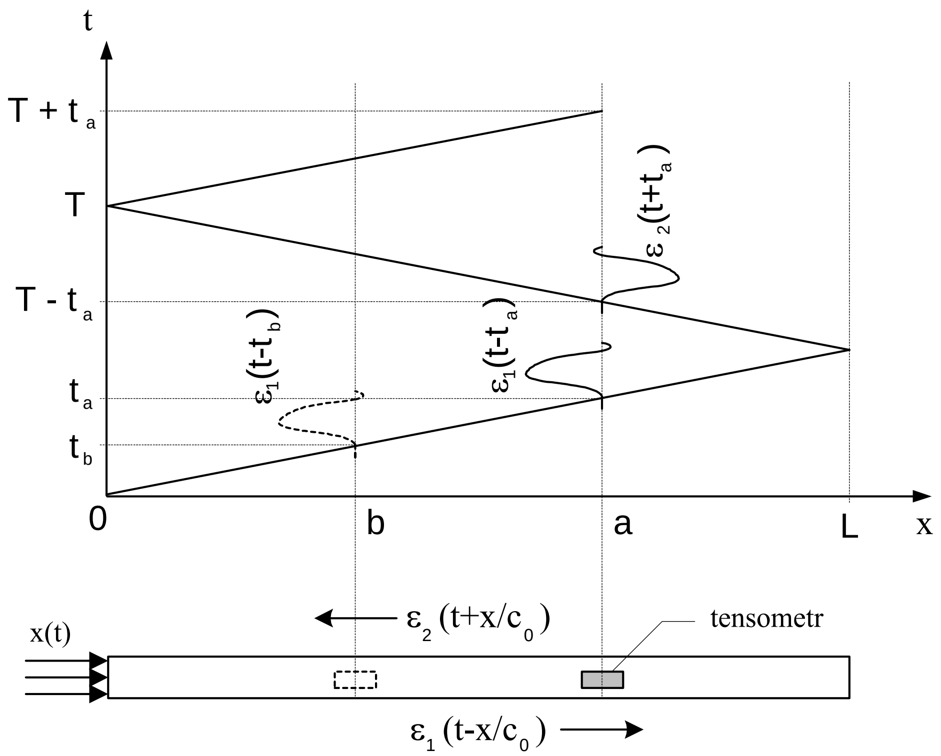


Figure1. Lagrangian diagram for longitudinal waves in a cylindrical bar of length L.

In Figure 1 the Lagrangian diagram is pictured, showing the propagation of elastic strain waves in a bar of length L. The strain gauges are mounted at distance a from the left end of the bar, on which the input is acting. The wave propagating in the right direction is designated ε_1 and the one propagating in the left direction - ε_2. Substituting x = a into equation (3) the following relation is obtained:

$$\varepsilon_A(t) = \varepsilon_1(t - t_a) + \varepsilon_2(t + t_a) \tag{4}$$

where $\varepsilon_A(t) = \varepsilon(a,t)$; $t_a = a / c_0$.

For the gauges to correctly process the strain waveforms, distance a should satisfy two conditions: $a \geq 20d$, where d = diameter of the bar and $2(L-a)/c_0 > \alpha$, where α = duration of the input. Meeting the first condition ensures the uniformity of the stress pattern in the cross-section of the bar, where the gauge is mounted, meeting the second one prevents the superposition of the incident and the first reflected pulse in the strain gauge.

In Figure 2, T denotes the time in which the wave runs twice through the bar $T = 2L / c_0$.
For $t < T - t_a$ the reflected wave does not propagate through the gauge. $\varepsilon_2(t + t_a) = 0$. From (4) arises:

$$\varepsilon_A(t) = \varepsilon_1(t - t_a) \tag{5}$$

For $T - t_a < t < T + t_a$, with $\alpha < T - 2t_a$, the incident wave does not propagate through the gauge. $\varepsilon_1(t - t_a) = 0$. From relation (4) follows:

$$\varepsilon_A(t) = \varepsilon_2(t + t_a) \tag{6}$$

For $t > T + t_a$ both the incident wave $\varepsilon_1(t - t_a)$ and the reflected wave $\varepsilon_2(t + t_a)$ are nonzero in general. The waveforms within this time range are not used in the reconstruction and dispersive distortions correction processes. In the sensor with one-point strain processing, waveforms are used, that are corresponding to non interfering waves: the incident $\varepsilon_1(t - t_a)$ and the first reflected one $\varepsilon_2(t + t_a)$. In comparison, in the two-point strain processing sensor, waveforms corresponding to the incident wave are used, when $t < T - t_a$, which are processed by the gauges at distance a $\varepsilon_A = \varepsilon_1(t - t_a)$ and also at distance b $\varepsilon_B = \varepsilon_1(t - t_b)$.

If the spectral transmittances of the incident and the first reflected waveform, with the delay of the waveforms neglected, are noted respectively as $\varepsilon_1(j\omega)$ and $\varepsilon_2(j\omega)$, than the spectral transmittance of the bar's section 2(L-a) is:

$$G_{2(L-a)}(j\omega) = -\frac{\varepsilon_2(j\omega)}{\varepsilon_1(j\omega)} \tag{7}$$

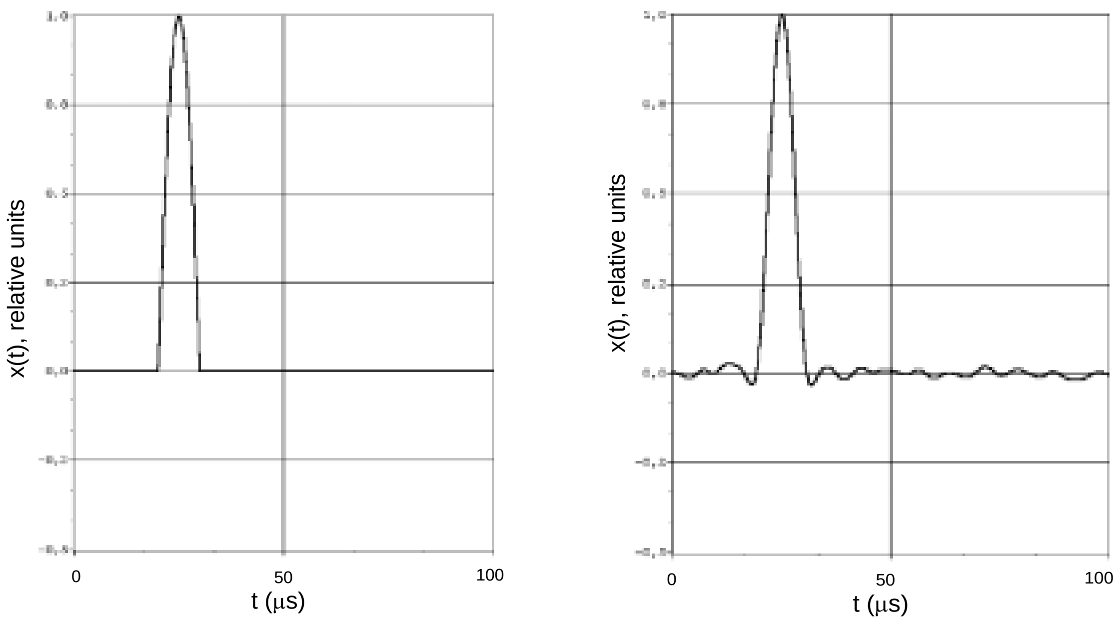
Assuming the homogeneity of the bar's material and also the condition, that 2(L-a) = a, it is possible to reconstruct the estimate of the spectrum $\widetilde{X}(j\omega)$ of the input waveform from relation:

$$\widetilde{X}(j\omega) = \frac{\varepsilon_1(j\omega)}{G_{2(L-a)}(j\omega)} K(j\omega, \gamma) \tag{8}$$

From formulas (7), (8), (9) arises the means of reconstructing impact force waveforms in the frequency domain, in which the one-point measurement of strain in a mechanical transducer is made us of. The choice of the regularization function K(jω, γ) as well as the selection of the regularization parameter γ can be performed as demonstrated in the paper (Kaczmarek 2001).

3 THE EXPERIMENT AND THE RESULTS

The frequency domain method of reconstruction, in which the Fourier Transform is utilized, as well as the one-point measurement of strain in a mechanical transducer, was used to reconstruct impact force waveforms originated as a result of impact of steel spheres against the front face of a steel bar. In the experiment a drawn bar was used, made of the type 40H steel, of 1,5 m length and 22 mm diameter. On the bar, at a distance of a = 1 m from its front face, on which the input function was acting, two semiconductor strain gauges of the type ESB-020-350, with the strain sensitivity coefficient K=155, were mounted and connected to form a Wheatstone bridge. The bridge was powered by 6 V direct voltage. To amplify the output of the bridge a INA 106 amplifier of K_u = 10V/V and frequency bandwidth B = 5 MHz was used. The recording of the voltage waveforms was carried out with the use of a TDS 340A digital oscilloscope. Before the experiment a reconstruction of simulated waveforms of the input function and the sensor output was performed. The input function waveform with the amplitude set by convention to 1 and of 10 μs duration is shown in Fig.3a.



a) b)

Figure 3. Simulated a) input function b) reconstructed input function.

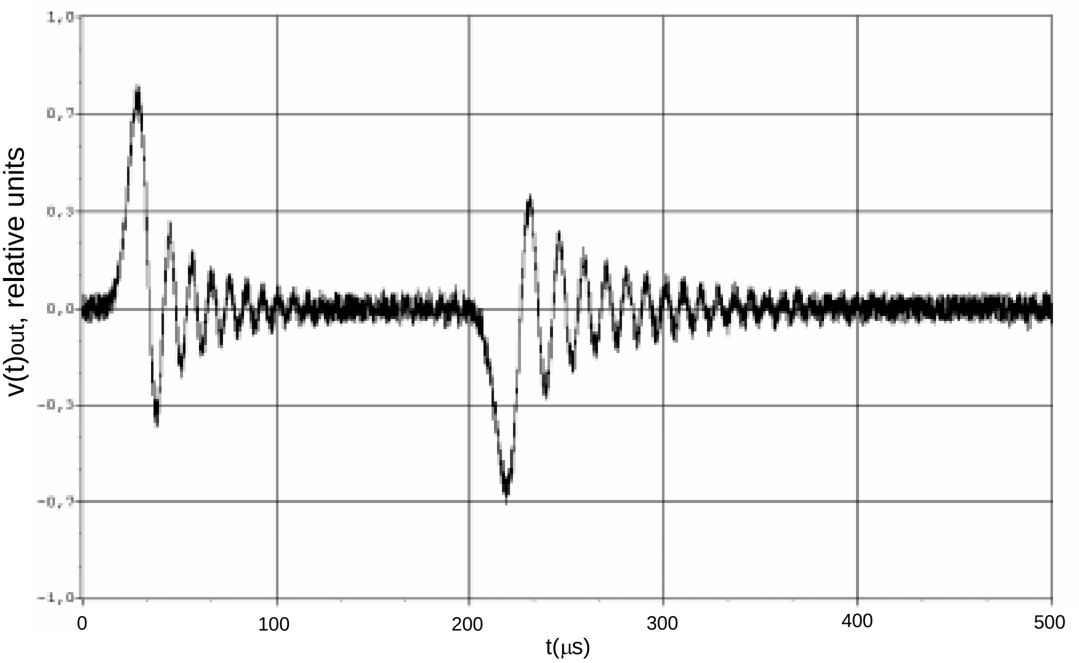


Figure 4. Simulated record of incident and first reflected pulses after travelling a distance of 1m in steel bar of diameter 22 mm.

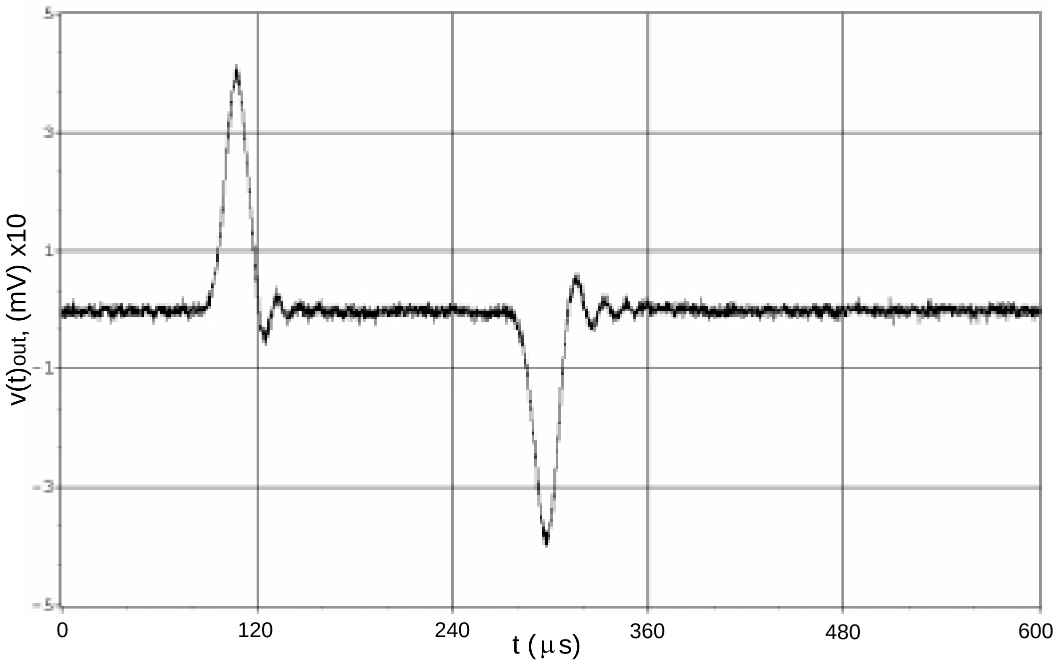


Figure 5. A record of incident and first reflected pulses generated in the steel bar by the impact of a steel sphere. Bar diameter = 22 mm, sphere diameter = 10 mm. impact velocity = 2m/s.

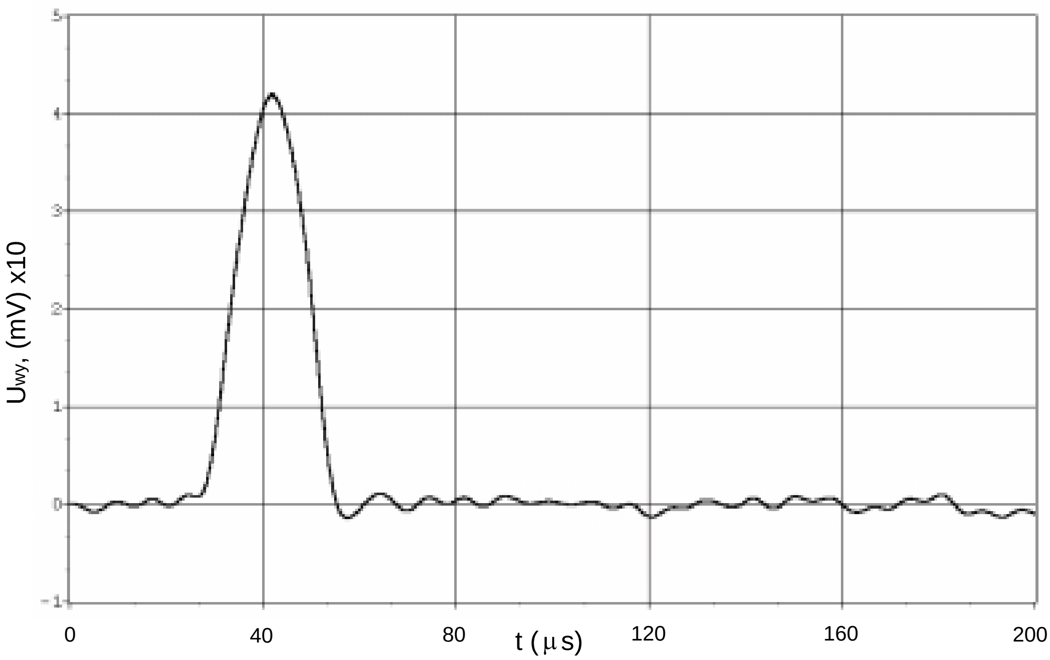


Figure 6. A reconstructed incident pulse.

In Figure 4 a simulated output signal from the sensor is shown, with the noise level at 0.05. In the signal, the incident and the first reflected pulse can be observed, that feature large oscillations following the trailing edges of the pulse, caused by the simulated dispersion. The input function waveform reconstructed using the transformation method is shown in Figure 3b. During the reconstruction, a regularisation filter was applied with the regularisation parameter $\gamma = 6{,}3 * 10^{-11}$. A high consistency between the input and the reconstructed waveform is apparent with respect to duration, amplitude and shape. In Fig. 5 a record of a waveform from the force sensor is presented, generated by the impact of a steel sphere, 10 mm in diameter. The impact velocity is 2 m/s. The effect of dispersion on the shape of the incident pulse and particularly the first reflected pulse in form of oscillations after the trailing edges of the pulses is evident. The duration and the amplitude value of the incident impact force are α_i = 33,5 μs and F_i = 960 N respectively. Figure 6 shows the impact force waveform reconstructed with the use of the transformation method. The duration and the amplitude value of the reconstructed impact force pulse equal α_r = 29,7 μs and 990 N respectively. These parameters calculated for the case mentioned above with the use of the Hertz Impact Theory have the following values: α = 30 μs and F = 995 N.

4 CONCLUSIONS

The means of reconstructing impulsive force waveforms, utilizing the Fourier Transform and the one-point measurement of strain inside a mechanical transducer, is a measure that allows to extend the application range of the Hopkinson bar in impulsive force measurements into the operating range that features dispersion and attenuation. In comparison with the two-point strain measurement inside a mechanical transducer the presented method requires a single channel signal conditioning circuit with the amount of information in the measuring circuit output retained. This makes it easier to reach the desired processing accuracy and in consequence the desired reconstruction accuracy.

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Kačmareks Z. Trieciena viļņa formas rekonstrukcija izmantojot frekvenču apgabala tehnoloģiju un punktveida deformācijas mērījumus.

Rakstā aplūkotā metode, kura ļauj atjaunot sākuma frekvences viļņa formu, izmantojot viena punktveida deformācijas mērījumus mehāniskā pārveidotājā. Tiek noteikta šī pārveidotāja spektrālā pārneses funkcija, kura nepieciešama rekonstrukcijas procesam. Šo funkciju nosaka pētot deformācijas viļņa izplatīšanos pārveidotājā: sakumā iedarbība, pirmais tās ietekmē atstarotais vilnis. Tiek formulēti ierobežojumi sākuma iedarbības ilgumam un deformācijas detektora novietojumam pārveidotājā. Ierobežojumi nodrošina augšminēto viļņu neatkarību. Tiek aprakstīti rezultāti viļņa sākuma formas rekonstrukcijai eksperimentam, kurā tērauda lodes iedarbojas uz tērauda brusas priekšpusi. Tiek salīdzināta iegūto rezultātu atbilstība Herca trieciena teorijai, ņemot vērā trieciena ilgumu un amplitūdu.

Качмарек З. Реконструкция начальной формы ударной волны с помощью технологии частотной области и измерений точечной деформации.

В статье рассмотрен метод, позволяющий восстановить начальную форму ударной волны, используя измерения точечной деформации в механическом преобразователе. Определена частотная передаточная функция преобразователя, которая необходима для процесса реконструкции. Указанную функцию получают путем изучения распространения волн деформации в преобразователе: начальное воздействие, первая отраженная волна деформации, вызванная этим воздействием. Сформированы ограничения на длительность и местоположение детектора деформации. Эти ограничения обеспечивают независимость вышеуказанных волн. Приведены результаты реконструкции для эксперимента, когда стальная сфера воздейстувет на переднюю часть стального бруса. Полученные результаты сопоставляются с теорией удара Герца в аспектах длительности и амплитуды воздействия.