Novel interpolation tools for creating hydrogeological models

A. Spalvins, J. Slangens, R. Janbickis, I. Lace
Environment Modelling Centre, Riga Technical University

Keywords: hydrogeology, modelling, software

ABSTRACT: The quality of a hydrogeological model (HM) depends not only on credibility of initial data, but also upon interpolation technologies applied to create HM. The Environment Modelling Centre (EMC) of the Riga Technical University has developed reliable interpolation tools. Theoretical ideas implemented into them are explained.

1 INTRODUCTION

The algebraic equation system (1) for HM (Fig. 1) is specified on the *xyz*-grid built of ($h\times h\times m$)-sized blocks; h and m are the constant plane size and a variable height of blocks

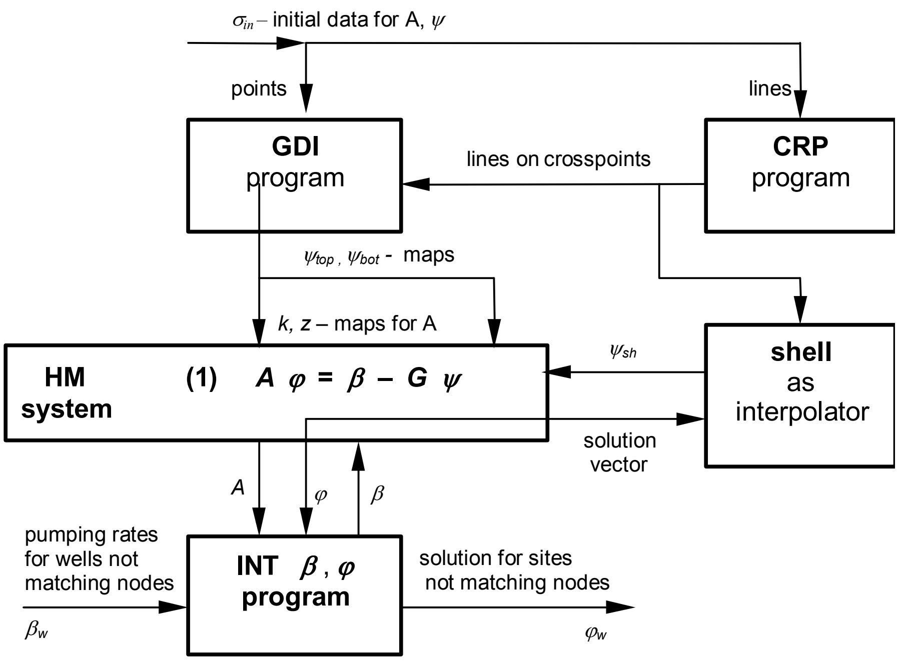


Fig. 1. The scheme of HM interpolation software.

In Fig. 1, φ is the solution vector (groundwater head) at nodes of the HM grid; it may be necessary to interpolate $\varphi \rightarrow \varphi_w$, at sites not matching the nodes.

Primary elements A, β, ψ of (1) are obtained by interpolation:

A – the symmetric matrix of the geological environment represented by a rectangular tiered *xy*-layer system of aquifers and interjacent aquitards; to obtain A, permeability k and elevation z-maps for each layer should be created;

ψ - the boundary head vector; ψ_{top}, ψ_{bot}, and ψ_{sh}-maps should be prepared for HM top, bottom and shell (four vertical sides of HM) surfaces, correspondingly;

G – the diagonal matrix (part of A) assembled of elements linking the nodes where φ must be found with the ones where ψ is given;

β - vector of water pumping rates; to obtain β, interpolation $\beta_w \rightarrow \beta$ is needed.

For these interpolations, special tools have been developed (see the scheme of Fig. 1):
geological data interpolation (GDI) program for creating the k, z, ψ_{top}, ψ_{bot}-maps (called σ-maps) and the program CRP (CRoss Point) for serving GDI and the shell [2, 5];
interpolation program INT β, φ for performing $\beta_w \rightarrow \beta$ and $\varphi \rightarrow \varphi_w$ [4];
to provide ψ_{sh}, the HM shell is acting as an interpolation device [3].

2 CRP PROGRAM

The CRP program turns [5] a raw data line l_{in} into standard l_σ and forms $l_\sigma \rightarrow l_c$ for the GDI program and the shell. The data lines l_{in} , l_σ, l_c lines are based on l.

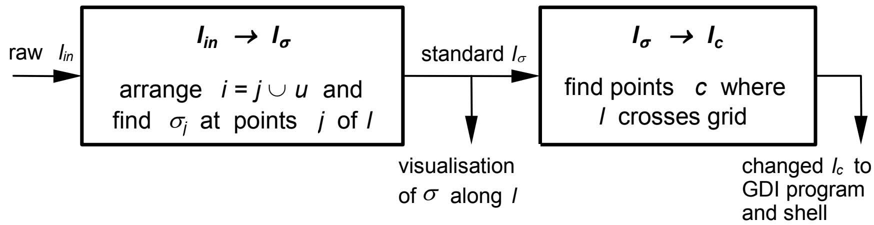


Fig. 2. The scheme of CPR program.

Vectorized l passes master points j where its direction changes. The points $j, j + 1$ are linked by the directed straight segment $l_{j,j+1}$, and l is the series:

$$l = \{ \theta_j \} = \{ x_j, y_j \},\ \ j = 1, 2, ..., J,\ \ d_{j,j+1} = \sqrt{(x_j - x_{j+1})^2 + (y_j - y_{j+1})^2},\ \ d_{1,J} = \sum_{j=1}^{J-1} d_{j,j+1}$$

where θ_j is coordinates of the j-th point; $d_{j,j+1}$ and $d_{1,J}$ are lengths of $l_{j,j+1}$ and l.

Raw $l_{in} = \{ l, \sigma_{in} \}$, $\sigma_{in} = \{ \theta_u , \sigma_u \}$, $u = 1, 2, \ldots, U$; u and j may not coincide.

Standard $l_\sigma = \{ \theta_i , \sigma_i \}$, $i = j \cup u = 1, 2, \ldots$, and interpolation on $\sigma_{in} \rightarrow \sigma_j$.

Changed $l_c = \{ \theta_c , \sigma_c \}$, $c = 1, 2, \ldots, C$, $d_{1,C} \rightarrow d_{1,J}$ if $h \rightarrow 0$, is based on points c where l crosses the grid and σ_c are treated by the GDI program as an irregular part of the system (1).

In GDI (Fig. 3), basic l is used for creating a mask μ. The line nullifies links v_{xy} of (3) crossed by l, and the xy-plane gets dissected into parts needed for μ [5].

Data lines including j and c-points are shown in the example demonstrating the GDI program.

3 GDI PROGRAM

The GDI program [2] provides a σ-map as the numerical solution of the Laplace's boundary problem (2) for a heterogeneous environment (Fig. 3)

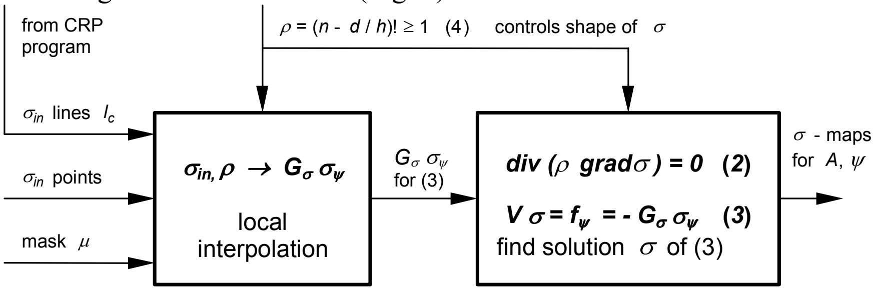


Fig. 3. The scheme of GDI program.

On the chosen xy-plane of (1), the algebraic equation system (3) approximates (2) where ρ is given by the factorial function (4) of Fig. 3 for positive rational numbers; V is the symmetric matrix of links $v_{xy} = \rho$; G_σ is the diagonal matrix (part of V) of elements connecting the nodes, where σ must be found, with the σ_ψ -nodes of the Dirichlet's boundary condition σ_ψ interpolated from σ_{in}.

The shape of σ is changed by ρ specified for σ_ψ -nodes. In (4), n controls the radius $n\times h$ of the $\rho > 1$ area; d is the distance from the σ_ψ -node to the one where $\rho > 1$ must be specified. When $n = 1$, $\rho = 1.0$, and then peaks of σ may appear at the σ_ψ -nodes. These peaks may be turned into dome-shaped when $n \geq 4$. If necessary, ρ may be controlled for each σ_ψ -node, or along any line chosen. Weighty advantage of (3) is upholding maximum/minimum and minimal energy principles. Then root sets of σ_{in} are minimal.

A round of GDI starts with local interpolation $\sigma_{in} \rightarrow \sigma_\psi$ [1] that involves pointwise and line data (e and c-data). Hence the e-data has the lowest rank, they are processed first:

$$\sigma_0 = \sum_{i=1}^{t} C_i\, \sigma_i\,, \qquad \sum_{i=1}^{t} C_i = 1.0\,, \qquad C_i = c_{i0} \,/\, \sum_{j=1}^{t} c_{j0}\,,$$

$$c_{i0} = (1 - |\,\xi_i\,| \,/\, h)\,(1 - |\,\eta_i\,| \,/\, h), \qquad c_{i0} = 0 \quad \text{if} \quad c_{i0} < 0.042\,. \tag{5}$$

The index 0 runs through $p = 1, 2, \ldots, N$ nodes of (3); σ_0 is found for the 0–th node, as the centre of the search area L_σ bounded by hyperbolic arcs; C_i and c_{i0} are total and partial weights of a source σ_i ; $\xi_i = x_i - x_0$, $\eta_i = y_i - y_0$ are local coordinates of σ_i. For an $h\times h$ block, (5) is illustrated by Fig. 4 and Fig. 5.

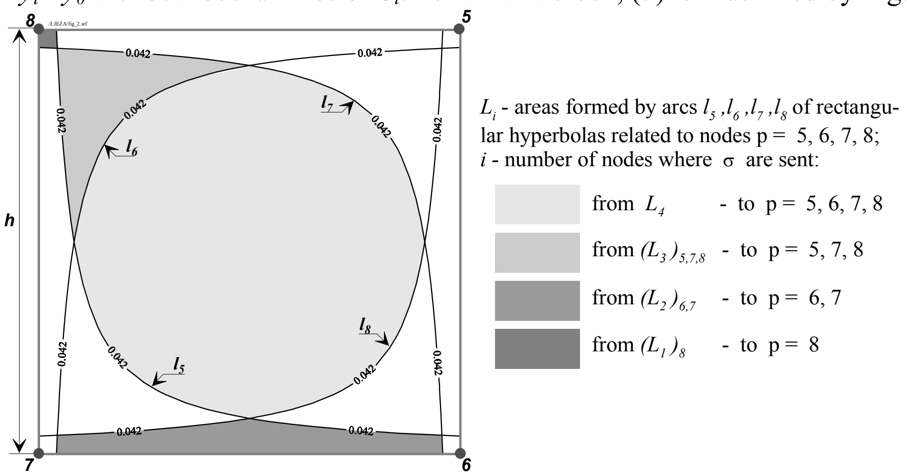


Fig. 4. Areas for point data search within an elementary $h\times h$ block if the optimal value $\lambda = 0.042$ (for the parameter of the hyperbolas-borderlines l_5, l_6, l_7, l_8) is applied.

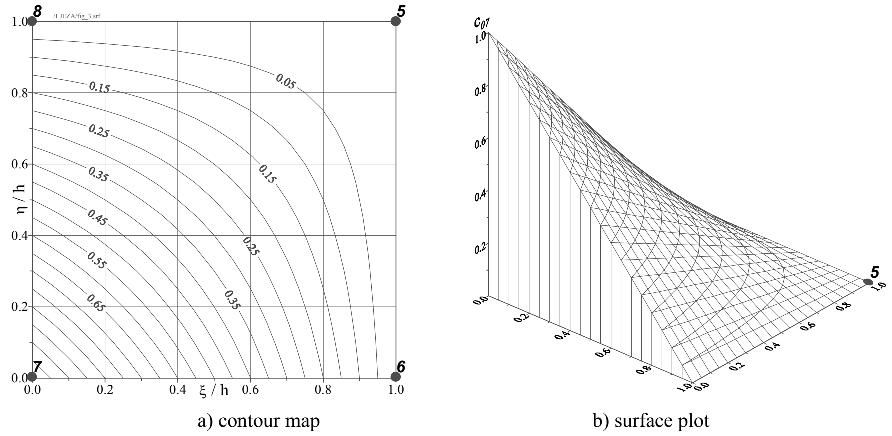


Fig. 5. Rectangular hyperbolas as the contours (isolines) of c_{07} on the grid block of Fig. 4.

The grid of (3) is controlled by the mask μ: if for the p-th node, $\mu_p = 1$ or 0 then σ_{in} are allowed or blocked here.

Commonly, c-data are carried by l_σ. For GDI, the CPR program finds { θ_c , σ_c }, c = 1, 2,… at the points where l_σ crosses the grid lines. These crosspoints are irregular nodes of (3). Local c-interpolation eliminates them, thus providing f_ψ and annihilating the result of (5) there, because l_c has higher rank upon e-data. Local conflicts of l_σ by different ranks may also cause serious errors. These ranks can be accounted for by repeating several rounds of (3). This way is convenient for detecting possible errors, and much simpler σ_{in} may be applied than if one tries to obtain σ at once. The surface created by GDI may include sharp edges and rows specified by l_σ.

The GDI program in action is demonstrated here by an example of creating a ψ_{top}-map.

4 INT β, φ PROGRAM

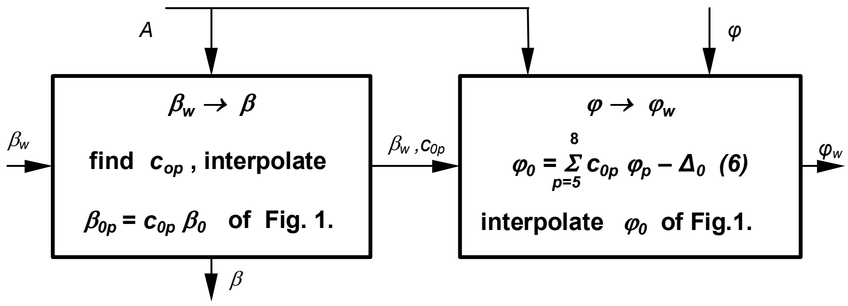
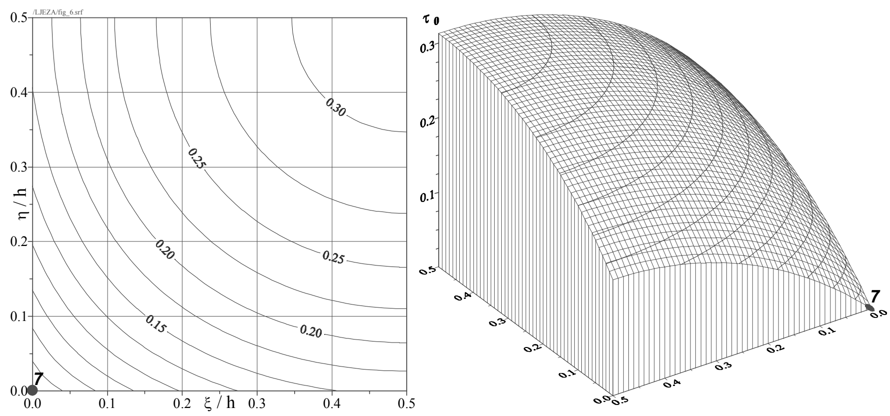


Fig. 6. The scheme of INT β, φ program.

When A and ψ of (1) are ready, $\beta_w \rightarrow \beta$ ($\beta_0 \rightarrow \beta_{op}$) is interpolated $\beta_{op} = c_{op}\, \beta_0$ to nodes p = 5, 6, 7, 8 of the grid block (Fig. 4) by the weights c_{op}.

The back-interpolation $\varphi \rightarrow \varphi_w$, at the 0-th site of Fig. 4, applies the formula (6) of Fig. 6, where $\Delta_0 = \Sigma\, \lambda_{0j}\, \beta_j$ ($j = 1,2,\ ...,\ t$) is the local depression caused by β_j sources via the weights λ_{0j}. Obtaining of c_{op} and λ_{0j} are explained in [4]. The principal element of λ_{0j} is the source function τ_0. Its contour map is shown in Fig. 7. The shape of τ_0 is close to a circle; in nodes, $\tau_0 = 0$.



a) contour map b) surface plot

Fig. 7. Contours of τ_0 on the quarter of the elementary grid block.

5 THE SHELL OF HM AS AN INTERPOLATOR

In many cases, conventional software or modeller is helpless to provide the right ψ_{sh}-distribution. The problem has been solved by converting the shell into an interpolation device by enlarging (10^3 - 10^5)-fold values of its links. The shell then acts like an almost ideally conducting shield computing missing values of ψ_{sh}, as a special portion of φ where no ψ components are fixed. This method may be applied in all kinds of modelling programs developed for HM.

6 CREATING BY GDI PROGRAM A FRAGMENT OF THE ψ_{TOP}-MAP FOR BERNAU HM, GERMANY

Materials of Bernau HM are taken from [6]. The example ψ_{top}-map presents the ground surface elevations with water pools included. The map is created in two rounds. For the first round, only pointwise σ_{in} of ground elevations are applied. The intermediate result $(\psi_{top})_1$ bears no impact of three pools and the railway embankment. These objects are accounted for during the second round by line σ_{in} (Fig. 9, Fig. 10).

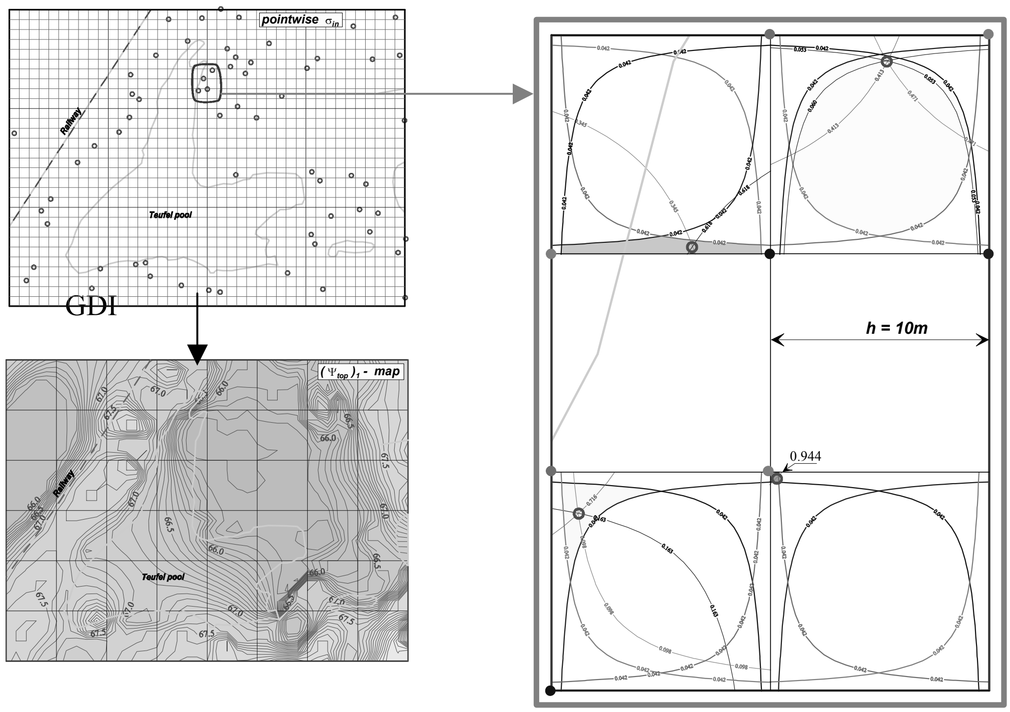


Fig. 8. The first round of GDI program.

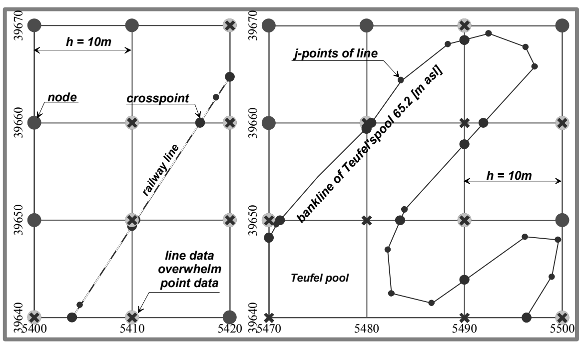


Fig. 9. Illustration of local interpolation for line σ_{in}.

Pointwise $(\psi_{top})_1$-map is masked to clear areas of pools where their banklines specify water elevations of pools as solution of (3).

Railway line itself clears its way through pointwise σ_{in} (Fig. 9).

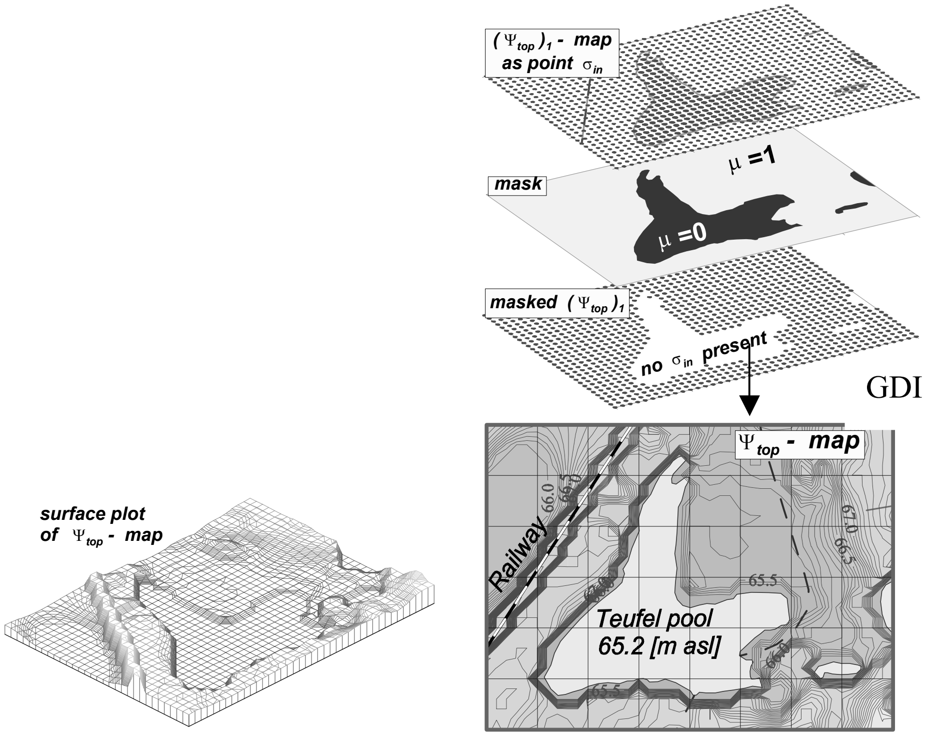


Fig. 10. The second (final) round of GDI program.

7 NEW THEORETICAL IDEAS IMPLEMENTED

- The method of local interpolation for pointwise data.
- To apply crosspoints of data lines as a part of the HM system.
- Using numerical solutions of boundary field problems for creating σ-maps by controlling the heterogeneousity parametre ρ.
- Complex σ–maps are obtained gradually by repeating interpolation rounds.
- Back-interpolation ($\varphi \rightarrow \varphi_w$) for irregular points of the HM body.
- Using the HM shell as an interpolation device.

The research has been financed by the latvian council of science

REFERENCES

1. Spalvins, A. & Slangens, J. 1995a. Local interpolation of geological environment data. *Proc. of International Seminar on "Environment Modelling". Boundary Field Problems and Computers. Riga – Copenhagen,* Volume 36, Part 1: 159-174.
2. *International Seminar on "Environment Modelling". Boundary Field Problems and Computers. Riga – Copenhagen,* Volume 36, Part 1: 175-192.

3. Spalvins, A. & Slangens, J. 1994. Numerical Interpolation of Geological Environment Data. *Proc. of Latvian - Danish Seminar on "Groundwater and Geothermal Energy". Boundary Field Problems and Computers. Riga – Copenhagen,* Volume 35, Part 2: 181-196.
4. Spalvins, A., Janbickis, R. & Slangens, J. 2000. Boundary Shells of Hydrogeological Models as Interpolation Devices, *Scient. Proc. of Riga Technical University in series "Computer Science". Boundary Field Problems and Computer Simulation. Riga,* Volume 4(42): 32-34.
5. Lace, I., Spalvins, A. & Slangens, J. 1995. Algorithms for Accounting Groundwater Discharge in the Regional Spalvins, A. & Slangens, J. 1995b. Updating of geological data interpolation programme. *Proc. of* Hydrogeological Model and Interpolation of Simulation Results at Observation Wells. *Proc. of International Seminar on "Environment Modelling". Boundary Field Problems and Computers, Riga – Copenhagen,* Volume 36, Part 1: 201-216.
6. Proceedings of ModelCARE 2002, Prague, Czech Republic, 17-20 June, 2002.
7. RTU raksts par Bernau 2002.g.

Aivars Spalvins, Dr.sc.ing.
Janis Slangens, Dr.sc.ing.
Romans Janbickis, M.sc.ing.
Inta Lace, M.sc.ing.
Riga Technical University, Faculty of Computer Science and Information Technology
Environment Modelling Centre,
Address: 1/4 Meza str., Riga, LV-1048, Latvia
Phone: +371 7089511
E-mail: emc@egle.cs.rtu.lv

Spalviņš A., Šlangens J., Janbickis R., Lāce I. Jauni interpolācijas līdzekļi hidroģeoloģisko modeļu veidošanai.
Hidroģeoloģisko modeļu (HM) kvalitāti nosaka ne tikai sākuma datu ticamība, bet arī interpolācijas tehnoloģija, kuru izmanto HM veidošanai. Rīgas Tehniskās universitātes Vides Modelēšanas centrs izstrādājis drošu interpolācijas līdzekļu sistēmu. Tajā ietilpst sekojošas savstarpēji saistītas komponentes: ģeoloģisko datu interpolācijas (GDI) programma, kura sagatavo galvenos HM sistēmas elementus; līniju datus GDI vajadzībām sagatavo krustpunktu (CRP) programma; speciāla programma radīta tādu ieejas un izejas datu interpolācijai, kuru novietojums atšķiras no HM režģa mezgliem; HM režģa čaula darbojas kā robežnoteikumu datu interpolators. Rakstā izklāstītas teorētiskās idejas, kuras izmantotas augšminēto līdzekļu izveidošanai.

Spalvins A., Slangens J., Janbickis R., Lace I. Novel interpolation tools for creating hydrogeological models.
The quality of a hydrogeological model (HM) depends not only on credibility of initial data, bat also upon interpolation technologies applied to create HM. The Environment Modelling Centre of the Riga Technical University has developed a system of reliable interpolation tools. It contains the following interdependent items: the geological data interpolation (GDI) program for obtaining principal elements of the HM system; line data for GDI is prepared by so called crosspoint (CRP) program; the special program has been developed for interpolation of input and output data for sites, not matching the grid nodes of HM; the shell of the HM grid acts as an interpolator for boundary conditions. In the paper, theoretical ideas implemented into these tools above are explained.

Спалвиньш А., Шлангенс Я., Янбицкис Р., Лаце И. Новые средства интерполяции для создания гидрогеологических моделей.
Качество гидрогеологических моделей (ГМ) определяется не только достоверностью исходных данных, а также технологией интерполяции, используемой для создания ГМ. Центром Моделирования окружающей среды из Рижского Технического Университета разработана надежная система средств интерполяции, в которую включены следующие взаймосвязанные средства: программа интерполяции геологических данных (GDI) которая создает основные элементы ГМ системы; для GDI информацию по заданным линиям подгатавливает программа точек пересечения (CRP); специальная программа создана для таких данных на входе и выходе системы, место расположения которых не совпадает с узлами сетки ГМ; оболочка сетки ГМ работает как интерполятор граничных условий. В данной статье, излагаются теоретические идеи, которые использованы для создания выше перечисленных средств.