ON FORMULAE FOR THE CHANGE IN IMPEDANCE

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1 INTRODUCTION

Formula for the change in impedance used in literature (see [Zaman, Gardner & Long, 1982], [Satveli, Moulder, Wang & Rose, 1996]) has the form

$$Z^{ind} = -\frac{(\sigma_F - \sigma)}{I^2} \int_{V_F} \vec{E} \cdot \vec{E}_F \, dV, \tag{1}$$

where V_F is the region of the flaw, σ_F and σ are the conductivities of the flawed and flawless regions, respectively, $\vec{E}_F$ is the amplitude electric field vector in the flawed region, $\vec{E}$ is the amplitude electric field vector in the same region in the absence of the flaw, I is the amplitude current vector density.

The displacement current is neglected in Eq. (1) as it is used in the problems of eddy current testing and in the case of harmonic oscillations of the external current with frequency ω (see [Antimirov, Kolyshkin & Vaillancourt, 1997]). In the present paper a new formula for Z^{ind} is obtained in more convenient for computations form:

$$Z^{ind} = \frac{\omega^2 (\sigma_F - \sigma)}{I^2} \int_{V_F} \vec{A} \cdot \vec{A}_F \, dV, \tag{2}$$

where $\vec{A}_F$ is the amplitude vector potential in the flawed region, $\vec{A}$ is the amplitude vector potential in the same region in the absence of the flaw (i.e. the case when all physical properties of region V_F are the same as the physical properties of the conducting region outside of region V_F), ω is the frequency.

The aim of this paper is to prove that the right-hand sides of Eqs. (1) and (2) coincide, i.e.

$$\int_{V_F} \vec{E} \cdot \vec{E}_F \, dV = -\omega^2 \int_{V_F} \vec{A} \cdot \vec{A}_F \, dV . \tag{3}$$

Note that the relationship between vectors $\vec{E}$ and $\vec{A}$ in the case of harmonic oscillations of the external current with frequency ω is given by (see [Antimirov, Kolyshkin & Vaillancourt, 1997]):

$$\vec{E} = -j\omega \vec{A} + \frac{1}{\tilde{k}_1^2} grad \; div \vec{A}, \tag{4}$$

where $\tilde{k}_1^2 = \mu_0 \mu (\sigma + j \varepsilon_0 \hat{\varepsilon} \omega)$ if the displacement current is taken into account and $\tilde{k}_1^2 = \mu_0 \mu \sigma$ if the displacement current is neglected, ε_0 and μ_0 are the electric and magnetic constants, respectively; $\hat{\varepsilon}$ and μ are the relative permittivity and relative magnetic permeability of the medium, respectively; $j=\sqrt{-1}$ is the imaginary unit.

It follows from Eq. (4) that Eq. (3) is correct if

$$div\,\vec{A}=0\,,\;\; div\,\vec{A}_F=0\,. \tag{5}$$

In fact Eq. (5) is only valid in the case of a homogeneous half-space as the conducting region and the external current located either on a single-turn coil or double conductor line in the plane parallel to the half-space. Eq. (5) is also valid if the flaw of the non-homogeneous half-space is a cylindrical body coaxial with a single-turn coil carrying the external current (see [Fastrickii, Antimirov & Kolyshkin, 1983], [Fastrickii, Antimirov & Kolyshkin, 1984]) or if the flaw is an infinitely long cylinder parallel to the double conductor line carrying the external current (see [Antimirov, Kolyshkin & Vaillancourt, 1994]). In all other cases, $div\,\vec{A}\neq 0$, $div\,\vec{A}_F\neq 0$ in region V_F. However, Eqs. (1), (2) and (3) are still true as it will be shown below.

It follows from Eqs. (3) and (4) that for a flaw situated in the arbitrary region V_F

$$\iiint_{V_F}[-j\omega\vec{A}\cdot\tilde{k}_1^2\,grad\,div\,\vec{A}_F-j\omega\vec{A}_F\cdot\tilde{k}_{1F}^2\,grad\,div\,\vec{A}+grad\,div\,\vec{A}\cdot grad\,div\,\vec{A}_F\,]\,dV=0\,, \tag{6}$$

where $\tilde{k}_{1F}^2=\mu_0\mu(\sigma_F+j\varepsilon_0\hat{\varepsilon}_F\omega)$. At first sight, assuming the continuity of the functions $\vec{A}$, $\vec{A}_F$, $grad\,div\,\vec{A}$, $grad\,div\,\vec{A}_F$, one may conclude that $grad\,div\,\vec{A}=0$, $grad\,div\,\vec{A}_F=0$ (using the known theorem: if a function $f(M)$ is continuous in a closed region V_F and for any region $\tilde{V}\subset V_F$ the formula $\iiint_{\tilde{V}} f(M)\,dV=0$ is valid, then $f(M)=0$ for all $M\in V_F$). However, it is not true. In fact, by changing the region V_F, the functions $\vec{A}$ and $\vec{A}_F$ are changed too. Therefore, Eq. (6) is also valid if $div\,\vec{A}\neq 0$, $div\,\vec{A}_F\neq 0$ in the region V_F. In the previous studies (see [Zaman, Gardner & Long, 1982], [Satveli, Moulder, Wang & Rose, 1996]) trying to prove formula (1) for impedance change, it was assumed that $div\,\vec{A}=0$ in Eq. (4). Besides, in [Zaman, Gardner & Long, 1982] was assumed that the scalar potential gives change in the static field only. That statement is not true. In [Satveli, Moulder, Wang & Rose, 1996] was suggested to use the Coulomb's gauge, i.e. $div\,\vec{A}=0$. At the same time, the authors use the following equation for the vector potential $\vec{A}$:

$$\Delta\vec{A}+k^2\vec{A}=\mu_0\mu\vec{I}^{ext}\,,\;\; k^2=-j\omega\sigma\mu_0\mu\,. \tag{7}$$

It is well known that Eq. (7) is not correct in this case. In fact, in the case of Coulomb gauge the equation for the vector potential is more complicated (see [Antimirov, Kolyshkin & Vaillancourt, 1997], p.10), and has the form

$$\Delta\vec{A}=\mu_0\mu\sigma\left(\nabla\varphi+\frac{\partial\vec{A}}{\partial t}\right)+\mu_0\varepsilon_0\mu\varepsilon\frac{\partial}{\partial t}\left(\nabla\varphi+\frac{\partial\vec{A}}{\partial t}\right)-\mu_0\mu\vec{I}^{e}\,, \tag{8}$$

where φ is the scalar potential. Moreover, even the authors of the present paper in their previous paper [Antimirov & Dzenite, 2002] mistakenly stated that Eq. (1) is not correct, using the fact that $div\,\vec{A}\neq 0$, $div\,\vec{A}_F\neq 0$ in the region V_F.

Note also that by taking into account the displacement current in this problem, the coefficient $\frac{\sigma_F - \sigma}{I^2}$ in Eqs. (1) and (2) is transformed into the coefficient

$$\frac{\sigma_F - \sigma}{I^2} + \frac{j\omega\varepsilon_0(\hat{\varepsilon}_F - \hat{\varepsilon})}{I^2}, \tag{9}$$

where $\hat{\varepsilon}_F$ and $\hat{\varepsilon}$ are the relative electric permittivity in the flawed and flawless regions, respectively.

2 THE PROOF OF FORMULA (1)

The proof is performed taking into account the displacement current. In the literature (see [Zaman, Gardner & Long, 1982], [Auld, Muennemann & Riaziat, 1984]) formula (1) is usually obtained without describing the source of external current. This fact makes difficulties for estimating the degree of mathematical basis of the formula. In the present paper, the emitter is located on a closed line described in the parametrical form in polar cylindrical coordinates (ρ, φ, z) by the equation:

$$\begin{cases} \rho = \rho(\varphi), \\ z = z(\varphi), \end{cases} \quad 0 \le \varphi \le 2\pi, \tag{10}$$

where $\rho(\varphi), z(\varphi)$ are prescribed functions, φ is the parameter. Equation describing any closed line can be written by using formula (10) and choosing the appropriate system of the rectangular coordinates (x, y, z).

Consider a sphere S_R of radius R with an interior arbitrary form closed surface S (see Fig.1).

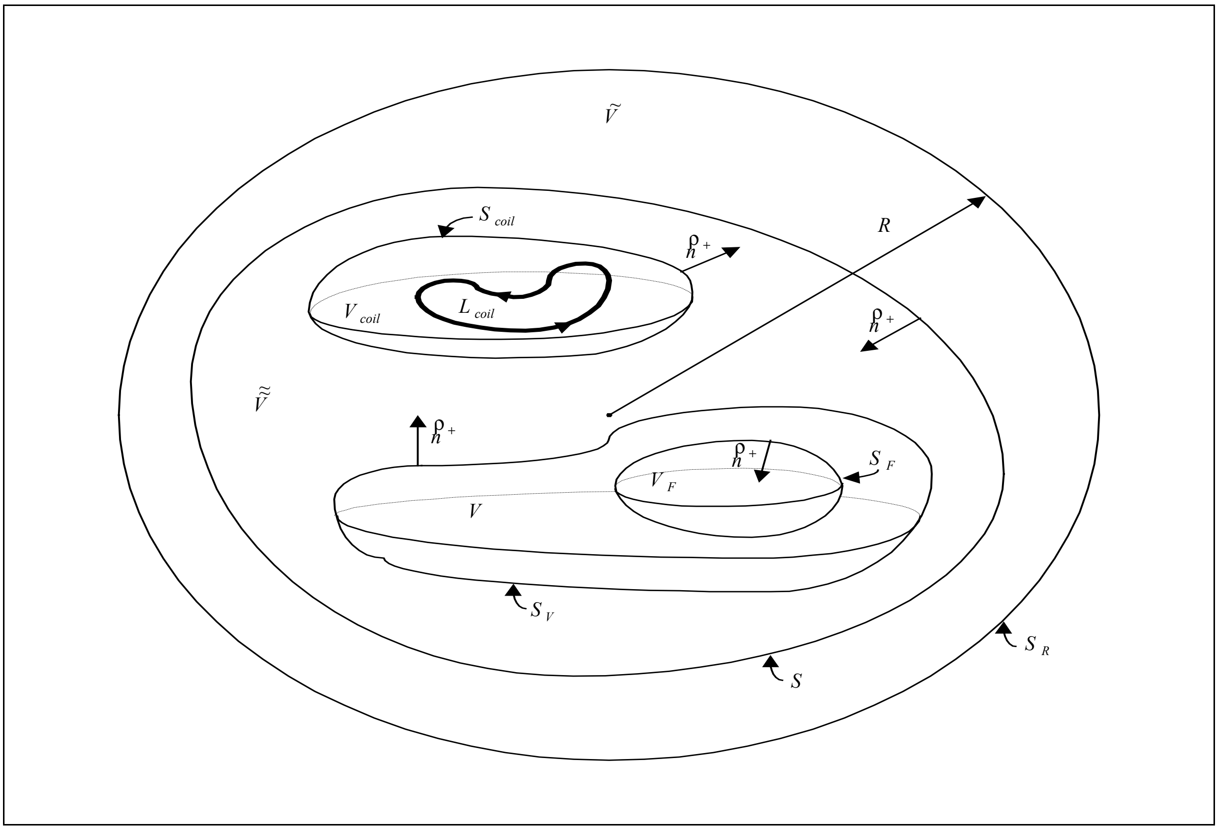

Fig.1. The disposition of the regions and closed surfaces.

The surface S covers the region containing a single-turn coil and a conducting medium. A closed surface S_{coil} bounds a region V_{coil} containing only a single-turn coil. A closed surface S_V bounds the region V containing the conducting medium with the conductivity $\sigma = const$ and the relative permittivity $\hat{\varepsilon} = const$ and a region V_F with the conductivity $\sigma_F = const$ and the relative permittivity $\hat{\varepsilon}_F = const$. The region V_F is bounded by a closed surface S_F. Finally, $\tilde{V}$ is a region bounded by surfaces S and S_R, and $\tilde{\tilde{V}}$ is a region bounded by surfaces S, S_{coil} and S_V.

In the case of harmonic oscillations of the external current with the frequency ω in the closed coil, Maxwell's equations for the complex-valued amplitude electric field vector $\vec{E}$ and the complex-valued amplitude magnetic field vector $\vec{H}$ have the following form (see [Antimirov, Kolyshkin & Vaillancourt, 1997]):

$$curl\,\vec{E} = -j\omega\mu_0\mu\vec{H}\,, \tag{11}$$

$$curl\,\vec{H} = (\sigma + j\varepsilon_0\hat{\varepsilon}\omega)\vec{E} + \vec{I}^e\,, \tag{12}$$

where $\vec{I}^e$ is the complex-valued amplitude external current vector density.

According to equation (10), one can write

$$\vec{I}^e = Ih(\rho,\varphi)\delta[\rho - \rho(\varphi)]\delta[z - z(\varphi)]\vec{e}_\tau\,, \tag{13}$$

where $\delta(x)$ is the Dirac delta function, $\vec{e}_\tau$ is a unit vector of the tangent to the line (10), I is the complex-valued amplitude current vector density. The coefficient $h(\rho,\varphi)$ in (13) has the form:

$$h(\rho,\varphi)=\frac{1}{\rho}\sqrt{\rho^2+[\rho'(\varphi)]^2+[z'(\varphi)]^2}\,. \tag{14}$$

This coefficient is chosen so that the triple integral of $\left|\vec{I}^e\right|$ over the all space is equal to the following constant:

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left|\vec{I}^e\right|dV = I\,L_{coil}=\sigma_{coil}\mathrm{E}_{coil}L_{coil}\,, \tag{15}$$

where σ_{coil} is the conductivity of the coil, L_{coil} is the length of the closed contour (10) with the current density $I=const$, $\mathrm{E}_{coil}L_{coil}$ is the electromotive force that is necessary for supporting the current of the density $I=const$ in the closed contour (10). It follows from Eq. (10) that the contour's length, L_{coil}, is equal to

$$L_{coil}=\int_0^{2\pi}\sqrt{[\rho(\varphi)]^2+[\rho'(\varphi)]^2+[z'(\varphi)]^2}\,d\varphi\,. \tag{16}$$

In order to prove Eq. (15), we substitute $\left|\vec{I}^e\right|$ given by Eq. (13) into the integral of Eq. (15). By using the main property of the delta function and using Eq. (16), we obtain

$$\begin{aligned}
&\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left|\vec{I}^e\right|dxdydz = I\int_0^{2\pi}d\varphi\int_0^{\infty}\frac{1}{\rho}\sqrt{\rho^2+[\rho'(\varphi)]^2+[z'(\varphi)]^2}\,\rho\,d\rho\times\\
&\times\int_{-\infty}^{+\infty}\delta[\rho-\rho(\varphi)]\,\delta[z-z(\varphi)]\,dz = I\int_0^{2\pi}\sqrt{\rho^2+[\rho'(\varphi)]^2+[z'(\varphi)]^2}\,d\varphi=\\
&=I\,L_{coil}=\sigma_{coil}\mathrm{E}_{coil}L_{coil}\,.
\end{aligned} \tag{17}$$

Thus, formula (15) is proved.

Explore the system of Eqs. (11)-(12) for two following cases: for the case when the flaw is absent, i.e. $\sigma_F=\sigma$ in the region V_F (by substituting $\vec{E}=\vec{E}_{abs}$, $\vec{H}=\vec{H}_{abs}$), and in the presence of the flaw (by substituting $\vec{E}=\vec{E}_F$, $\vec{H}=\vec{H}_F$). Then assuming that the external current vector density $\vec{I}^e$ is the same in both cases and it is defined by Eq. (13), one obtains

$$-curl\,\vec{E}_{abs}=j\omega\mu_0\mu\vec{H}_{abs}\,, \tag{18}$$

$$curl\,\vec{H}_{abs}=\tilde{k}^2_{abs}\vec{E}_{abs}+\vec{I}^e\,, \tag{19}$$

$$-curl\,\vec{E}_F=j\omega\mu_0\mu\vec{H}_F\,, \tag{20}$$

$$curl\,\vec{H}_F=\tilde{k}^2_F\vec{E}_F+\vec{I}^e\,, \tag{21}$$

where

$$\tilde{k}^2_{abs}=\begin{cases}\sigma+j\varepsilon_0\hat{\varepsilon}\omega, & M(x,y,z)\in V,\\ j\varepsilon_0\hat{\varepsilon}\omega, & M(x,y,z)\notin V,\end{cases} \tag{22}$$

$$\tilde{k}_F^2 = \begin{cases} \sigma_F + j\varepsilon_0 \hat{\varepsilon}_F \omega, & M(x,y,z) \in V_F, \\ \tilde{k}^2, & M(x,y,z) \notin V_F. \end{cases} \tag{23}$$

In the above, $\vec{E}_{abs}$ and $\vec{H}_{abs}$ are the solutions of Eqs. (18)-(19) such that:

1) the tangent components of the vectors $\vec{E}_{abs}$ and $\vec{H}_{abs}$ are continuous on the surface S_V (see [Antimirov, Kolyshkin & Vaillancourt, 1997]);
2) vectors $\vec{E}_{abs}$ and $\vec{H}_{abs}$ satisfy the radiation condition at infinity (see [Tihonov & Samarsky, 1972]).

Similarly, $\vec{E}_F$ and $\vec{H}_F$ are the solutions of Eqs. (20)-(21) such that:

1) the tangent components of the vectors $\vec{E}_F$ and $\vec{H}_F$ are continuous on the surfaces S_F and S_V;
2) vectors $\vec{E}_F$ and $\vec{H}_F$ satisfy the radiation condition at infinity.

In order to prove formula (1) we use the Lorentz reciprocity theorem (see [Antimirov, Kolyshkin & Vaillancourt, 1997]). By taking the scalar product of Eq. (19) with $\vec{E}_F$ and of Eq. (20) with $\vec{H}_{abs}$, and by summing both products, one obtains

$$\vec{E}_F \cdot curl\,\vec{H}_{abs} - \vec{H}_{abs} \cdot curl\,\vec{E}_F = \tilde{k}_{abs}^2\, \vec{E}_{abs} \cdot \vec{E}_F + \vec{I}^e \cdot \vec{E}_F + j\omega\mu_0\mu\, \vec{H}_{abs} \cdot \vec{H}_F . \tag{24}$$

From

$$div(\vec{E}_F \times \vec{H}_{abs}) = \vec{H}_{abs} \cdot curl\,\vec{E}_F - \vec{E}_F \cdot curl\,\vec{H}_{abs}, \tag{25}$$

and Eq. (20), it follows

$$-div(\vec{E}_F \times \vec{H}_{abs}) = \tilde{k}_{abs}^2\, \vec{E}_{abs} \cdot \vec{E}_F + \vec{I}^e \cdot \vec{E}_F + j\omega\mu_0\mu\, \vec{H}_{abs} \cdot \vec{H}_F . \tag{26}$$

Interchanging subscripts abs and F in Eq. (26) (i.e. doing the similar operations with Eqs. (18) and (21)), we obtain

$$-div(\vec{E}_{abs} \times \vec{H}_F) = \tilde{k}_F^2\, \vec{E}_F \cdot \vec{E}_{abs} + \vec{I}^e \cdot \vec{E}_{abs} + j\omega\mu_0\mu\, \vec{H}_F \cdot \vec{H}_{abs} . \tag{27}$$

Subtracting Eq. (26) from Eq. (27) yields

$$div(\vec{E}_F \times \vec{H}_{abs} - \vec{E}_{abs} \times \vec{H}_F) = (\tilde{k}_F^2 - \tilde{k}_{abs}^2)\vec{E}_{abs} \cdot \vec{E}_F - \vec{I}^e \cdot (\vec{E}_F - \vec{E}_{abs}) . \tag{28}$$

I. Integration of Eq. (28) over the region $\tilde{V}$ bounded by the closed surfaces S_R and S yields

$$\int_{\tilde{V}} div(\vec{E}_F \times \vec{H}_{abs} - \vec{E}_{abs} \times \vec{H}_F)\, dV = (\tilde{k}_F^2 - \tilde{k}_{abs}^2) \int_{\tilde{V}} \vec{E}_{abs} \cdot \vec{E}_F\, dV - \int_{\tilde{V}} \vec{I}^e \cdot (\vec{E}_F - \vec{E}_{abs})\, dV . \tag{29}$$

Since $\tilde{k}_{abs}^2 - \tilde{k}_F^2 = 0$ and $\vec{I}^e = 0$ in the region $\tilde{V}$ (see Eqs. (13), (22), (23)), the right-hand side of Eq. (29) is equal to zero in the region $\tilde{V}$. The left-hand side is transformed using the Gauss' divergence theorem and taking into account that the boundary of region $\tilde{V}$ consists of two closed surfaces S_R and S (see Fig.1). As a result, we obtains

$$\left[\oint_{S_R} + \oint_{S}\right](\vec{E}_F \times \vec{H}_{abs} - \vec{E}_{abs} \times \vec{H}_F) \cdot \vec{n}^+ dS = 0, \tag{30}$$

where $\vec{n}^+$ is a unit vector of the external normal to the boundary of the region $\tilde{V}$. We assume that as $R \to \infty$, the integrand in Eq. (28) tends to zero faster than R^{-2}. Since the surface S_R is a sphere of radius R, we have

$$\lim_{R\to\infty} \oint_{S_R} (\vec{E}_F \times \vec{H}_{abs} - \vec{E}_{abs} \times \vec{H}_F) \cdot \vec{n}^+ dS = 0. \tag{31}$$

Thus, it follows from Eq. (31) that

$$\oint_{S} \vec{R} \cdot \vec{n}^+ dS = 0, \tag{32}$$

where

$$\vec{R} = \vec{E}_F \times \vec{H}_{abs} - \vec{E}_{abs} \times \vec{H}_F. \tag{33}$$

II. Integrating Eq. (28) over the region $\tilde{\tilde{V}}$ bounded by three closed surfaces S, S_{coil} and S_V, using the Gauss' divergence theorem and taking into account that in the region $\tilde{\tilde{V}}$ the right-hand side of Eq. (28) is equal to zero, we obtain

$$\left(\oint_{S} + \oint_{S_{coil}} + \oint_{S_V}\right) \vec{R} \cdot \vec{n}^- dS = 0, \tag{34}$$

where $\vec{n}^- = -\vec{n}^+$ is a unit vector of the external normal to the boundary of the region $\tilde{\tilde{V}}$. It follows from Eqs. (32) and (34) that

$$\oint_{S_{coil}} R \cdot \vec{n}^- dS = -\oint_{S_V} \vec{R} \cdot \vec{n}^- dS = \oint_{S_V} \vec{R} \cdot \vec{n}^+ dS. \tag{35}$$

III. Integrating Eq. (28) over the region V_{coil} bounded by the closed surface S_{coil}, then using the Gauss' divergence theorem and taking into account that in this region $\vec{I}^e \neq 0$ and $\vec{I}^e$ is defined by Eq. (13), one gets

$$\oint_{S_{coil}} \vec{R} \cdot \vec{n}^+ dS = -I \iiint_{V_{coil}} \delta[\rho - \rho(\varphi)]\delta[z - z(\varphi)]\, h(\rho,\varphi)\vec{e}_\tau \cdot \vec{E}^{ind} dV, \tag{36}$$

where

$$\vec{E}^{ind} = \vec{E}_F - \vec{E}_{abs}. \tag{37}$$

The right-hand side of Eq. (36) is transformed using the main property of delta function:

$$\iiint_{V_{coil}} \delta[\rho - \rho(\varphi)]\delta[z - z(\varphi)]\, h(\rho,\varphi)\vec{e}_\tau \cdot \vec{E}^{ind} dxdydz =$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \delta[\rho - \rho(\varphi)]\, h(\rho,\varphi)\vec{e}_\tau \cdot \vec{E}^{ind}\Big|_{z=z(\varphi)} dxdy, \tag{38}$$

where

$$h(\rho,\varphi)=\frac{1}{\rho}\sqrt{\rho^2+[\rho'(\varphi)]^2+[z'(\varphi)]^2}\,.$$

Introducing the polar cylindrical coordinates $x=\rho\cos\varphi$, $y=\rho\sin\varphi$, $dxdy=\rho\, d\rho d\varphi$ into Eq. (38) yields

$$F\equiv\int_0^{2\pi}d\varphi\int_0^{\infty}\delta[\rho-\rho(\varphi)]\,h(\rho,\varphi)\,\vec{e}_\tau\cdot\vec{E}^{ind}\Big|_{z=z(\varphi)}\rho\, d\rho=$$

$$=\int_0^{2\pi}\vec{E}^{ind}(\rho(\varphi),z(\varphi))\,h(\rho(\varphi),\varphi)\,\vec{e}_\tau\,\rho(\varphi)\,d\varphi\,. \tag{39}$$

However,

$$\vec{e}_\tau h(\rho(\varphi),\varphi)\rho(\varphi)d\varphi=\vec{e}_\tau dl=d\vec{l}\,, \tag{40}$$

where $d\vec{l}$ is a such vector that its module is equal to the differential of the length of the line arc and it is directed along the tangent to this line. Thus, it follows from Eq. (39) that

$$F=\oint_{L_{coil}}\vec{E}^{ind}\cdot d\vec{l}=-Z^{ind}I\,, \tag{41}$$

where Z^{ind} is the change in impedance due to a flaw situated in the region V_F (see [Antimirov, Kolyshkin & Vaillancourt, 1997]). Consequently, Eq. (36) has the following form

$$\oint_{S_{coil}}\vec{R}\cdot\vec{h}^{+}dS=I^2Z^{ind}\,. \tag{42}$$

IV. Integrating Eq. (28) over the region V bounded by two closed surfaces S_V and S_F, using the Gauss' divergence theorem and taking into account that the right-hand side of Eq. (28) is equal to zero in this region, we obtain

$$\oint_{S_V}\vec{R}\cdot\vec{h}^{+}dS+\oint_{S_F}\vec{R}\cdot\vec{h}^{+}dS=0\,. \tag{43}$$

It follows from Eqs. (35), (42) and (43) that

$$-I^2Z^{ind}=\oint_{S_V}\vec{R}\cdot\vec{h}^{+}dS=-\oint_{S_F}\vec{R}\cdot\vec{h}^{+}dS\,. \tag{44}$$

V. Finally, integrating Eq. (28) over the region V_F, using the Gauss' divergence theorem and taking into account that $\vec{I}^{e}=0$ and $\tilde{k}_F^2-\tilde{k}_{abs}^2=\sigma_F-\sigma+j\omega\varepsilon_0(\hat{\varepsilon}_F-\hat{\varepsilon})$ in this region, we obtain

$$-\oint_{S_F}\vec{R}\cdot\vec{h}^{+}dS=[(\sigma_F-\sigma)+j\omega\varepsilon_0(\hat{\varepsilon}_F-\hat{\varepsilon})]\int_{V_F}\vec{E}_{abs}\cdot\vec{E}_F\,dV\,. \tag{45}$$

The final formula follows from Eqs. (44) and (45):

$$Z^{ind}=-\frac{1}{I^2}[(\sigma_F-\sigma)+j\omega\varepsilon_0(\hat{\varepsilon}_F-\hat{\varepsilon})]\int_{V_F}\vec{E}_{abs}\cdot\vec{E}_F\,dV\,. \tag{46}$$

3 THE PROOF OF FORMULA (2) FOR THE CASE OF AN ARBITRARY

Let us consider two arbitrary functions $u(M)=u(x,y,z)$, $v(M)=v(x,y,z)$, which are continuous together with their second derivatives, in the region V bounded by some closed surfaces $S_1, S_2, \ldots, S_m$. Using the Green's formula, one gets

$$\iiint_V (u\Delta v - v\Delta u)\,dV = \left(\oint_{S_1} + \oint_{S_2} + \ldots + \oint_{S_m} \right)\left(u\frac{\partial v}{\partial n} - v\frac{\partial u}{\partial n} \right) dS\,, \tag{47}$$

where $\vec{n}$ is the external normal vector of the region V. Formula (47) is also valid for two vector functions $\vec{u}(M)$ and $\vec{v}(M)$.

Let $\vec{A}_{abs}(M)$ be the vector potential in the absence of the flaw, $\vec{A}_F(M)$ be the vector potential in the presence of the flaw. The vectors $\vec{A}_{abs}$ and $\vec{A}_F$ satisfy the following equations (see [Antimirov, Kolyshkin & Vaillancourt, 1997]):

$$\Delta\vec{A}_{abs} + \tilde{k}^2_{abs}\vec{A}_{abs} = -\mu_0\mu\vec{I}^{\,e}\,, \tag{48}$$

$$\Delta\vec{A}_F + \tilde{k}^2_F\vec{A}_F = -\mu_0\mu\vec{I}^{\,e}\,, \tag{49}$$

where $\tilde{k}^2_{abs}$, $\tilde{k}^2_F$ and $\vec{I}^{\,e}$ are defined by Eqs. (22), (23) and (13), respectively. Vectors $\vec{E}_{abs}$, $\vec{H}_{abs}$, $\vec{E}_F$ and $\vec{H}_F$ are expressed in terms of the vectors $\vec{A}_{abs}$ and $\vec{A}_F$ by using the following expressions (see [Antimirov, Kolyshkin & Vaillancourt, 1997]):

$$curl\,\vec{A}_{abs} = \mu_0\mu\vec{H}_{abs}\,, \qquad \vec{E}_{abs} = -j\omega\vec{A}_{abs} + \frac{1}{\mu_0\mu}\frac{1}{\tilde{k}^2_{abs}}\,grad\,div\vec{A}_{abs}\,, \tag{50}$$

$$curl\,\vec{A}_F = \mu_0\mu\vec{H}_F\,, \qquad \vec{E}_F = -j\omega\vec{A}_F + \frac{1}{\mu_0\mu}\frac{1}{\tilde{k}^2_F}\,grad\,div\vec{A}_F\,. \tag{51}$$

The Green's formula (47) for the vectors $\vec{A}_{abs}$ and $\vec{A}_F$ in the region $\tilde{V}$ bounded by the closed surfaces S_R and S (see Fig.1), has the form:

$$\iiint_{\tilde{V}} (\vec{A}_{abs}\Delta\vec{A}_F - \vec{A}_F\Delta\vec{A}_{abs})\,dV = \left(\oint_{S_R} + \oint_S \right)\left(\vec{A}_{abs}\frac{\partial\vec{A}_F}{\partial n} - \vec{A}_F\frac{\partial\vec{A}_{abs}}{\partial n} \right) dS\,. \tag{52}$$

By substituting $\Delta\vec{A}_{abs} = -\tilde{k}^2_{abs}\vec{A}_{abs} - \mu_0\mu\vec{I}^{\,e}$ of Eq. (48) and $\Delta\vec{A}_F = -\tilde{k}^2_F\vec{A}_F - \mu_0\mu\vec{I}^{\,e}$ of Eq. (49) into the left-hand side of Eq. (52), and taking into account that $\vec{I}^{\,e}=0$, $\tilde{k}^2_{abs}=\tilde{k}^2_F$ in the region $\tilde{V}$, one can see that the left-hand side of Eq. (52) is equal to zero. In the limit $R\to\infty$, the integral over S_R tends to zero. Consequently, it follows from Eq. (52) that

$$\oint_S \left(\vec{A}_{abs}\frac{\partial\vec{A}_F}{\partial n} - \vec{A}_F\frac{\partial\vec{A}_{abs}}{\partial n} \right) dS = 0 \text{ as } R\to\infty\,. \tag{53}$$

Formula (53) is completely equivalent to formula (32). Therefore, further proof of formula (2) is completely equivalent to that of formula (1). Consequently,

$$Z^{ind} = \frac{\omega^2}{I^2}[\sigma_F - \sigma + j\omega\varepsilon_0(\hat{\varepsilon}_F - \hat{\varepsilon})] \iiint_{V_F} \vec{A}_{abs} \cdot \vec{A}_F \, dV . \tag{54}$$

Formula (54) can be generalised for the case of m flaws present in regions $V_{F1}, V_{F2}, \ldots, V_{Fm}$ bounded by surfaces $S_1, S_2, \ldots, S_m$, respectively. The regions $V_{F1}, V_{F2}, \ldots, V_{Fm}$ have the conductivities $\sigma_1, \sigma_2, ..., \sigma_m$ and the relative permittivities $\hat{\varepsilon}_1, \hat{\varepsilon}_2, ..., \hat{\varepsilon}_m$, respectively. Using the same proof as above, instead of formula (53) one obtains

$$Z^{ind} = \sum_{k=1}^{m} \frac{\omega^2}{I^2}[\sigma_k - \sigma + j\omega\varepsilon_0(\hat{\varepsilon}_k - \hat{\varepsilon})] \iiint_{V_{Fk}} \vec{A}_{abs} \cdot \vec{A}_{Fk} \, dV, \tag{55}$$

where $\vec{A}_{Fk}$ is the vector potential in region V_{Fk} with the conductivity σ_k and under the condition that all other regions of the flaw, $V_{F1}, V_{F2}, \ldots, V_{Fk-1}, V_{Fk+1}, \ldots, V_{Fm}$, have the conductivities equal to $\sigma_1, \sigma_2, ..., \sigma_{k-1}, \sigma_{k+1}, \ldots, \sigma_m$, respectively.

4 CONCLUSIONS

A new formula for the change in impedance is obtained for the case of a closed emitter of arbitrary form located above a conducting region with an arbitrary form flaw. The formula has the form of a triple integral over the region of the flaw of scalar product of two vector potentials: the vector potential in the flaw and the vector potential in the same region in the absence of the flaw. It is strictly proved that the new simple formula is equivalent to the previous formula used in literature. However, the previous authors used this new simple formula in their applications without a correct basis. The new formula is generalized for the case of m arbitrary flaws located in the conducting region. The displacement current is taken into account.

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Antimirovs M., Dzenīte I. Par impedances izmaiņu aprēķināšanas formulām.
Iegūta jauna analītiska formula impedances izmaiņu aprēķināšanai, kuru izmanto nesagraujošās kontroles problēmās. Pierādījumam izmantota Grīna formula, atšķirībā no iepriekšējiem darbiem, kuros, lai iegūtu pazīstamu literatūrā formulu, izmanto Lorenca teorēmu. Jaunā formula impedances izmaiņu aprēķināšanai iegūta trīskāršā integrāļa formā pa apgabalu, kas satur defektu, no divu vektoru potenciālu skalārā reizinājuma: vektora potenciālu defektā un vektora potenciālu tajā pašā apgabalā, gadījumā, kad defekta nav. Līdzīgai formulai, kas iegūta iepriekš, ir elektriskā lauka vektoru skalārā reizinājuma trīskāršā integrāļa forma. Stingri pierādīts, ka jaunā vienkāršākā formula ir ekvivalenta formulai, kas izmantota literatūrā.

Antimirov M. Ya., Dzenite I. A. On formulae for the change in impedance.
A new exact analytical formula for the impedance change used in non-destructive testing problems is derived. The derivation is based on the Green's formula in contrast with the previous studies that used Lorentz theorem for obtaining the formula known in literature. The new formula for the impedance change has the form of a triple integral of scalar product of two vector potentials: the vector potential in the flaw and the vector potential in the same region in the absence of the flaw over the region containing the flaw. The similar formula obtained earlier by previous authors has the form of a triple integral of scalar product of amplitude electric field vectors. It is strictly proved that the new simple formula is equivalent to the previous formula used in literature.

Антимиров М., Дзените И. О формулах для вычисления изменений в импедансе.
Получена новая аналитическая формула для вычисления изменений в импедансе, используемом в задачах неразрушающего контроля. Доказательство основано на формуле Грина, в отличие от предыдущих работ, в которых для получения формулы известной в литературе, используется теорема Лоренца. Новая формула для вычисления изменений в импедансе имеет форму тройного интеграла по области, содержащей дефект, от скалярного произведения двух векторных потенциалов: векторного потенциала в дефекте и векторного потенциала в той же области, но при условии, что дефект отсутствует. Сходная формула, полученная ранее предыдущими авторами, имеет форму тройного интеграла от скалярного произведения векторов электрического поля. Строго доказано, что новая более простая формула эквивалентна предыдущей формуле, используемой в литературе.