ELECTRODYNAMICAL AND OPTIMIZATION PROBLEMS OF OVAL THREE-PHASE HEAVY CURRENT LINES

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INTRODUCTION

Quick development of technological civilization is accompanied by growing demand for electric power. This imposes the need for searching new methods and equipment for high power transmission. Three-phase heavy-current lines (i.e. power busways) are reckoned among such transmission devices. They are used for power transmitting in middle voltage (up to 20 kV) or high voltage ranges, and current values ranging to several kA. They are made as cylindrical symmetric systems (three conductors distributed each 120º in a shield of circular cross-section), as plane systems (three conductors located aside, with or without the shield), or as three individually shielded busways. As an insulating medium may be used: the air, sulphur hexafluoride SF_6, or a solid component made of epoxy-resins.

An important factor for the process of production of material goods (inclusive of designing the electric equipment) is saving of raw materials and energy. Proper determination of electrodynamical parameters of a system is of important practical meaning when analysing electrical equipment of any type. Designing and carrying out power conductors for higher and higher current and voltage values give rise to the need of accurate description of electromagnetic, dynamical, and thermal phenomena. Knowledge of quantitative and qualitative relationship between a comprehensive set of electrodynamical parameters (power losses, temperatures, electrodynamical forces, voltage gradients) and structural parameters (materials, shapes, and dimensions) is necessary for carrying out the optimization process, taking into account the costs of manufacture and exploitation. Analysis in the above mentioned scope is feasible with the use of numerical methods and computer technics. The problems related to determination of electrodynamical parameters may be considered both with the use of analytical and numerical methods. The tendency aimed at joining these both methods, i.e. developing algorithms making use of analytical solutions in one area and the numerical one in another, is also natural. Such an approach remarkably speeds up the numerical calculation and reduces the requirements related to computer equipment.

The work presents an algorithm of electrodynamical calculation related to heavy-current lines operating in a cylindrical symmetric arrangement. As an insulating medium the air is applied. Distribution of current density occurring in the phase conductors and induced in the shield was determined with the help of the method of integral equations. Knowledge of the current density distribution enabled calculating of active power losses in the system and, afterwards, the distribution of conductor and shield temperatures. Maximal electrodynamic forces and voltage gradients in the systems have been also analyzed.

The presented electrodynamical considerations and calculations are used to optimization of the shape and cross-section of three-phase shielded air-insulated heavy current lines in two variants - determination of dimensions with regard to:

– minimal consumption of structural material during manufacture of conductors and shield,

– minimal power losses and consumption of structural material of the busway.

In order to achieve high effectiveness of the optimization procedure a modified genetic algorithm was applied.

DESCRIPTION OF THE SYSTEM AND METHODS OF DETERMINATION OF ELECTRODYNAMICAL PARAMETERS

The analysis was carried out for a three-phase shielded heavy current line composed of three phase conductors (Fig. 1) each of cross-section S_c , located symmetrically every 120° in a shield of inner radius R_3 and outer radius R_4. As a basis for determining the electrodynamical parameters of the system serves the distribution of current density J of the phase conductor. For calculation purposes a cylindrical system of coordinates (r,φ,z) has been used.

The electrodynamical calculations (particularly considering heat phenomena) presented here are used for searching optimal design of heavy-current busways. The electrodynamical parameters (serving for formulating of optimization constraints) are calculated many times within the optimization algorithm. Therefore, the calculation process should be simplified to the utmost degree, keeping at the same time its due accuracy. Taking into account the nature of the system and the properties of electrodynamic interaction occurring therein, the computation was confined to a planar linear system (the system is symmetrical and the length of the conductors significantly exceeds cross dimensions).

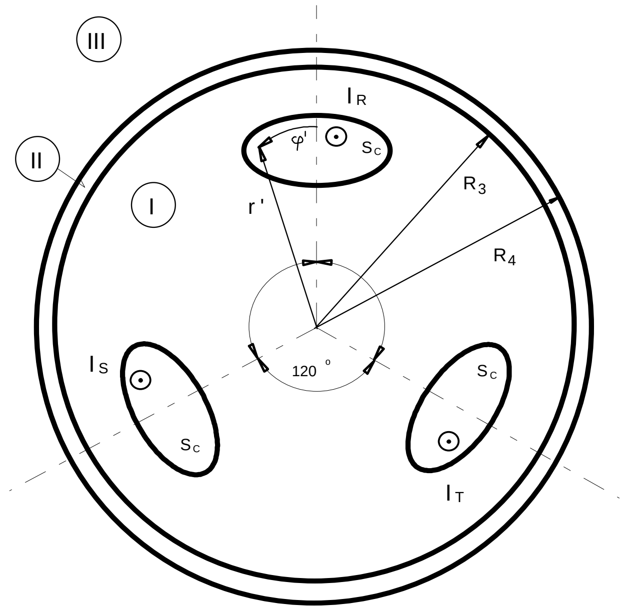

Fig. 1. Analyzed system with marked particular areas

Current density distribution in current busways and its shield

The magnetic vector potential **A**(r,φ,z) in the considered 2D system has only one component in *z*-axis direction, and depends only on the coordinates (r,φ), i.e. **A**(r,φ,z) = $\mathbf{1}_z$ A(r,φ), satisfying in particular areas (Fig. 1) the following relationships [2]:

- in the I-area (inside the shield), i.e. for $0 \le r \le R_3$:

$$\mathbf{A}_I(r,\varphi)=\mathbf{A}_1(r,\varphi)+\mathbf{A}_2(r,\varphi) \tag{1}$$

The potential $\mathbf{A}_1$ (r,φ) included in the relationship is a result of the currents flowing through the phase conductors, and may be expressed by the formula:

$$\mathbf{A}_1(\mathrm{r},\varphi)=\frac{3}{4\pi}\mu_O \int_{S_C} J(\mathrm{r}',\varphi')\sum_{i=1}^{\infty}\left[a_i \sin i(\varphi-\varphi')+b_i \cos i(\varphi-\varphi')\right]\frac{x^i}{i} r'dr'd\varphi' \qquad (2)$$

The potential $\mathbf{A}_2$ (r,φ) is a result of the currents induced in the shield and satisfies the Laplace equation:

$$\nabla^2 \mathbf{A}_2(\mathrm{r},\varphi)=0 \qquad (3)$$

- in the II-area (material of the shield), i.e. for $R_3 \le r \le R_4$:

$$\nabla^2 \mathbf{A}_{II}(\mathrm{r},\varphi) = j\omega\,\mu_O\,\mu_S\gamma_S \mathbf{A}_{II}(\mathrm{r},\varphi) \qquad (4)$$

- in the III-area (outside the shield), i.e. for $r \ge R_4$:

$$\nabla^2 \mathbf{A}_{III}(\mathrm{r},\varphi) = 0 \qquad (5)$$

The following definitions are related to the above relationships:
- S_C - cross-section area of a phase conductor,
- J - current density of the R-phase,
- ω - pulsation (ω = 2πf),
- μ_O - magnetic permeability of vacuum,
- μ_S - relative magnetic permeability of the shield material,
- γ_S - conductivity of the shield material,
- the coefficients x, a_i and b_i take the values [2,8]:

$$x=\begin{cases}\frac{r}{r'} & \text{for } r\le r' \\ \frac{r'}{r} & \text{for } r\ge r'\end{cases}, \quad a_i=\begin{cases}0 & \text{for } i=3l \\ j & \text{for } i=3l-1 \\ -j & \text{for } i=3l-2\end{cases}, \quad b_i=\begin{cases}0 & \text{for } i=3l \\ 1 & \text{for } i\ne 3l\end{cases} \qquad (6)$$

for l = 1, 2, 3, ...

Moreover, the following boundary conditions should be met at the boundaries of particular areas:
- for r = R_3:

$$A_I(r,\varphi) = A_{II}(r,\varphi)$$
$$H_{I\varphi}(r,\varphi) = H_{II\varphi}(r,\varphi)$$

- for r = R_4: (7)

$$A_{II}(r,\varphi) = A_{III}(r,\varphi)$$
$$H_{II\varphi}(r,\varphi) = H_{III\varphi}(r,\varphi)$$

Solving the equations (1)÷(4) and (6) satisfying the boundary conditions (7) leads to the expression for magnetic vector potential in analyzed areas.

Distribution of current density J(r,φ) in the phase conductor is derived from an approximate solution of integral equations [2,8] obtained in result of the use of known relationships for the electromagnetic field $\mathbf{E} = -j\omega\mathbf{A}$ and $\mathbf{J} = \gamma\mathbf{E}$:

$$J(r,\varphi)-J(r_O,\varphi_O)+\frac{3}{4\pi}j\omega\mu_O\gamma_C\int_S J(r',\varphi')\left[K(r',\varphi',r,\varphi)-K(r',\varphi',r_O,\varphi_O)\right]dr'\,d\varphi'=0 \tag{8}$$

$$\int_S J(r',\varphi')r'\,dr'\,d\varphi'=I \tag{9}$$

where (r_o,φ_o) is a reference point, γ_C is conductivity of conductor material, I – intensity of the current flowing in the R-phase, while $K(r',\varphi',r,\varphi)$ – is a kernel of the integral equation determined by the relationship [2]:

$$\begin{aligned}K(r',\varphi',r,\varphi) = &\sum_{i=1}^{\infty}\left[a_i \sin i(\varphi-\varphi') + b_i \cos i(\varphi-\varphi')\right]\frac{x^i}{i}r' + \\ &+\sum_{i=1}^{\infty}\left[F_{1i}\sin i(\varphi-\varphi')+F_{2i}\cos i(\varphi-\varphi')\right](r\,r')^i\,r\end{aligned} \tag{10}$$

The coefficients occurring in equation (10) are described in [8].

It results from symmetry of the system that distribution of current density of two remaining phase conductors (S and T) is the same as in R but shifted by +120° and -120°, respectively.

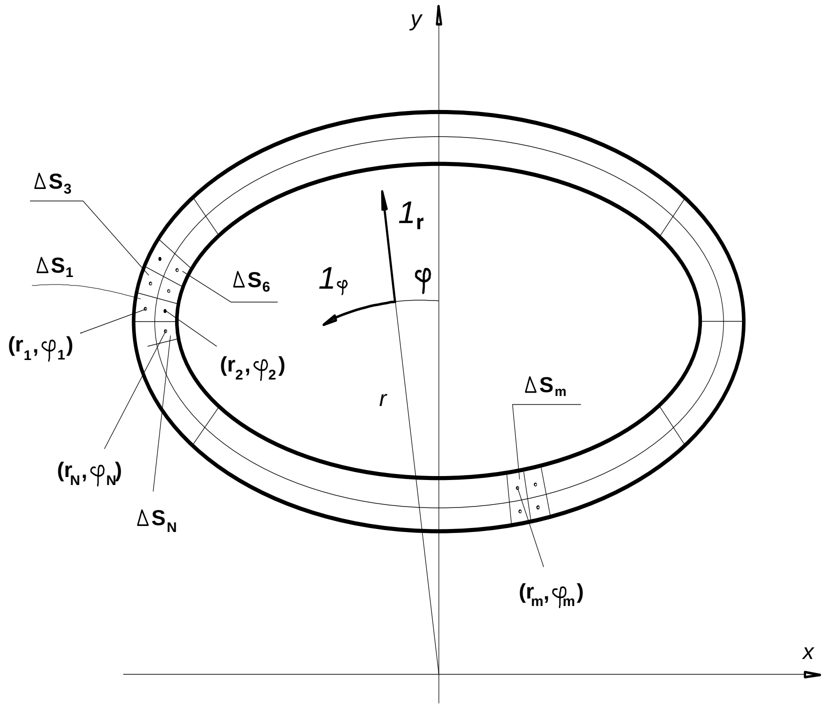

Fig. 2. Division of cross-section area of the phase conductor into N parts

The presented system of integral equations may be solved in approximate manner using a moment method, being a variation of Ritz method [2,6]. In order to apply this method the cross-section S of the conductor is divided into N elements (Fig. 2) of the areas ΔS_m (with m=1,2,...,N). The discreti-zation has been made making use of the geometry of conductor cross-section. The section is in the form of an oval ring.

The oval (dealt in the present paper) is a figure composed of four circular sectors. Therefore, the sectors of the oval ring (being the sectors of appropriate circular rings) are divided in angular and radial directions into elementary ΔS_m surfaces for which coordinates of their centres are determined.

Current density is expanded in functional space:

$$J(r,\varphi) = \sum_{m=1}^{N} J_m f_m \tag{11}$$

while $J_m = J(r_m,\varphi_m) = \text{const}$ (for m = 1, 2, ..., N) and f_m are base functions defined as follows:

$$f_m = \begin{cases} 1 & \text{for } \Delta S_m \\ 0 & \text{for the remaining elements} \end{cases} \tag{12}$$

In result, J_m is an approximate value of current density in a ΔS_m – element.

Making use of the moment method the system of integral equations is then replaced by a system of N linear algebraic equations in the form

$$\begin{bmatrix} l_{1,1} & l_{1,2} & l_{1,3} & \cdots & l_{1,N} \\ l_{2,1} & l_{2,2} & l_{2,3} & \cdots & l_{2,N} \\ l_{3,1} & l_{3,2} & l_{3,3} & \cdots & l_{3,N} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ l_{N-1,1} & l_{N-1,2} & l_{N-1,3} & \cdots & l_{N-1,N} \\ \Delta S_1 & \Delta S_2 & \Delta S_3 & \cdots & \Delta S_N \end{bmatrix} \begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ \vdots \\ J_{N-1} \\ J_N \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ I \end{bmatrix} \tag{13}$$

where:

$$l_{m,n} = \delta_{m,n} - \delta_{N,n} + \frac{3}{4\pi} j\omega\mu_O\gamma_C \int_{\Delta S_m} \left[K(r',\varphi',r_m,\varphi_m) - K(r',\varphi',r_N,\varphi_N)\right] dr'\, d\varphi' \tag{14}$$

for: m = 1,2,3,...,N-1 and n = 1,2,3,...,N, $\delta_{m,n}$ is Kronecker delta, $K(r',\varphi',r,\varphi)$ - a kernel of the integral equation defined in the form (10).

Solution of the system (13) provides approximate values J_m (for m = 1, 2, 3,...., N) of current density in particular elements of cross-section ΔS_1, ΔS_2, ..., ΔS_N. Total current I flowing in the phase conductor may be considered as a set of m conductors transmitting the currents, $I_m = J_m \Delta S_m$, (m = 1,2,...,N).

Distribution of the current induced in the shield is obtained with the use of the relationships of electromagnetic field: $\mathbf{E} = -j\omega\mathbf{A}$ and $\mathbf{J} = \gamma\mathbf{E}$. Substituting the relationship for magnetic vector potential within the shield area [2] one finally obtains:

$$J_S(r,\varphi) = -\frac{3}{4\pi} j\omega\mu_O\gamma_S \int_S J(r',\varphi') \left\{ \sum_{i=1}^{\infty} \left[G_{1i} K_i(kr) + G_{2i} I_i(kr)\right] \sin i(\varphi - \varphi') + \sum_{i=1}^{\infty} \left[G_{3i} K_i(kr) + G_{4i} I_i(kr)\right] \cos i(\varphi - \varphi') \right\} r^{-1} r'\, dr'\, d\varphi' \tag{15}$$

Coefficients occurring in the above equation are determined in [1,2,8].

Calculation of active power losses in phase conductors and the shield

Knowledge of approximate distribution of the current density vector enables determining the value of active power losses in conductors and shield. The losses of active power in phase conductors P_C (falling to unit length) may be determined e.g. from the Joule law [1,8]:

$$P_C = \frac{3}{\gamma_C} \int_S \left| J(r',\varphi') \right|^2 r'\,dr'\,d\varphi' \tag{16}$$

Taking into account the approximate distribution of current density, the relationship takes a form:

$$P_C = \frac{3}{\gamma_C} \sum_{m=1}^{N} \left| J_m(r_m,\varphi_m) \right|^2 \Delta S_m \tag{17}$$

and the losses of active power in shield P_s:

$$P_S = \frac{1}{\gamma_S} \int_{Ss} \left| J_S(r',\varphi') \right|^2 r'\,dr'\,d\varphi' \tag{18}$$

Calculation of temperatures in phase conductors and the shield

Knowledge of the active power losses and the distribution of power density emitted in the conductors and in the shield is necessary for determining thermal conditions of the system.

Distribution of power density output in the shield is expressed by the relationship:

$$\rho(r,\varphi) = \frac{\left| J_S(r',\varphi') \right|^2}{\gamma_S} \tag{19}$$

Results of many calculations made for aluminium shield of 3-mm thickness have shown that $\rho(r,\varphi)$ is a function strongly dependent on φ and symmetrically distributed every 120° [2], while the variable r only slightly affects ρ.

Total thermal power emitted from phase conductors dissipates radially, approximately uniformly [2,4,9]. At the outer surface of the shield the following boundary condition is met:

$$\frac{P_C}{2\pi R_3} = -\lambda_S \frac{\partial T}{\partial r} \quad \text{for} \quad r = R_3 \tag{20}$$

Inside the shield the temperature meets Poisson equation:

$$\nabla^2 T(r,\varphi) = -\frac{\rho(r,\varphi)}{\lambda_S} \tag{21}$$

From outer surface of the shield the thermal power is emitted to the environment in result of convection and radiation:

$$\lambda \frac{\partial \mathrm{T}(\mathrm{r},\varphi)}{\partial \mathrm{r}} = -\alpha_{\mathrm{CR}} \left[\mathrm{T}(\mathrm{r},\varphi) - \mathrm{T_O}\right] \qquad \text{for} \qquad \mathrm{r} = \mathrm{R}_4 \tag{22}$$

Temperature excess above that of the environment meets the Poisson equation [2,3,9]:

$$\lambda_{\mathrm{C}} \frac{\partial^2 \mathrm{T}(\mathrm{l})}{\partial \mathrm{l}^2} = -\mathrm{p}(\mathrm{l}) \qquad \text{, where} \qquad \mathrm{p}(\mathrm{l}) = \frac{1}{\mathrm{g}} \sum_{\mathrm{j}=1}^{\mathrm{n}} \mathrm{P_j}\, \delta(\mathrm{l} - \mathrm{l_{mj}}) - \frac{2\alpha}{\mathrm{g}} \mathrm{T}(\mathrm{l}) \tag{23}$$

is a difference between the power emitted from the branch through n inner sources, and the power carried away to the environment with constant α coefficient, $\delta(l - l_{mj})$ is delta function, g is the thickness of conductor wall.

Methods of solving the above questions and determining the coefficients occurring in the equations are discussed in detail in the works [2,3,4,8,9].

Determination of electrodynamical forces acting in the system

The force $\Delta\mathbf{F}$ acting on unit length of the conductor carrying the current of density $\mathbf{J}$ of the cross-section $\Delta\mathbf{S}$ placed in the field of induction $\mathbf{B}$ is determined by the relationship [8]:

$$\Delta\,\mathbf{F} = (\,\mathbf{J} \times \mathbf{B}\,)\, \Delta\mathbf{S} \tag{24}$$

In the system of cylindrical coordinates the magnetic induction $\mathbf{B}$ expressed in terms of vector magnetic potential is equal to:

$$\mathbf{B} = \mathrm{rot}\ \mathbf{A} = \mathbf{1}_\mathrm{r}\, \mathrm{B_r} + \mathbf{1}_\varphi\, \mathrm{B}_\varphi \tag{25}$$

with:

$$\mathrm{B_r} = \frac{1}{\mathrm{r}} \frac{\partial \mathbf{A}}{\partial \varphi} \qquad , \qquad \mathrm{B}_\varphi = -\frac{\partial \mathbf{A}}{\partial \mathrm{r}} \tag{26}$$

Substituting the above relationships into the formula (24) gives [8]:

$$\Delta\mathbf{F} = \mathbf{1}_\mathrm{r}\, \Delta\mathrm{F_r}\ (\mathrm{r},\varphi,\mathrm{t}) + \mathbf{1}_\varphi\, \Delta\mathrm{F}_\varphi\ (\mathrm{r},\varphi,\mathrm{t}) \tag{27}$$

where:

$$\Delta\mathrm{F_r} = \mathrm{J}\, \Delta\mathrm{S} \frac{1}{\mathrm{r}} \frac{\partial \mathrm{A}}{\partial \varphi} \qquad , \qquad \Delta\mathrm{F}_\varphi = \mathrm{J}\, \Delta\mathrm{S} \frac{\partial \mathrm{A}}{\partial \mathrm{r}} \tag{28}$$

Distribution of the current density and, further on, distribution of the magnetic induction are calculated in the complex domain. The above expressions in the time-form, take the following shape:

$$\Delta\mathrm{F}_\varphi (\mathrm{r},\varphi,\mathrm{t}) = 2\left|\,\mathrm{J}\,\right| \Delta\mathrm{S} \left|\,\mathrm{B_r}\right| \sin\,(\omega\, \mathrm{t} + \psi_\mathrm{J})\, \sin\,(\omega\, \mathrm{t} + \psi_\mathrm{r}) \tag{29}$$

$$\Delta\mathrm{F_r} (\mathrm{r},\varphi,\mathrm{t}) = -\,2\left|\,\mathrm{J}\,\right| \Delta\mathrm{S} \left|\,\mathrm{B}_\varphi\right| \sin\,(\omega\, \mathrm{t} + \psi_\mathrm{J})\, \sin\,(\omega\, \mathrm{t} + \psi_\varphi) \tag{30}$$

In the presented relationships ψ_J, ψ_r, ψ_φ are initial phase shifts of J , B_r , B_φ , respectively. Induction values B_r and B_φ occurring in the point defined by the coordinates (r_m, φ_m), i.e. in the middle of the elementary cross-section ΔS_m leading the current $I_m = J_m \Delta S_m$, are determined as a sum of inductions caused by all elementary conductors of the system, except for the induction of the given elementary conductor (nevertheless, the induction caused by two other phases and currents induced in the shield are taken into consideration).

$$B_r(r_m,\varphi_m)=\sum_{\substack{n=1\\n\neq m}}^{N} B_{rn}(r_m,\varphi_m)+\frac{3\mu_0}{4\pi}\frac{1}{r_m}J_m\int_{\Delta S_m}\left\{-\frac{j}{3}+\right.$$
$$\left.+\sum_{i=1}^{\infty} i\left[F_{1i}\cos i(\varphi_m-\varphi')-F_{2i}\sin i(\varphi_m-\varphi')\right](r\,r')^i\right\}r'\,dr'\,d\varphi' \tag{31}$$

$$B_\varphi(r_m,\varphi_m)=\sum_{\substack{n=1\\n\neq m}}^{N} B_{\varphi n}(r_m,\varphi_m)-\frac{3\mu_0}{4\pi}\frac{1}{r_m}J_m\int_{\Delta S_m}\left\{\frac{1}{3}+\right.$$
$$\left.+\sum_{i=1}^{\infty} i\left[F_{1i}\sin i(\varphi_m-\varphi')+F_{2i}\sin i(\varphi_m-\varphi')\right](r\,r')^i\right\}r'\,dr'\,d\varphi' \tag{32}$$

Substituting the above expressions into the formulas of the forces provides appropriate equations for the forces acting at particular elements of the conductor cross-section as time functions and their average values falling to a cycle. The force values fall to unit length:
- in time-dependent form:

$$\Delta F_\varphi(r_m,\varphi_m,t)=2\left|J_m\right|\Delta S_m\left|B_r(r_m,\varphi_m)\right|\sin(\omega t+\psi_J)\sin(\omega t+\psi_r) \tag{33}$$

$$\Delta F_r(r_m,\varphi_m,t)=-2\left|J_m\right|\Delta S_m\left|B_\varphi(r_m,\varphi_m)\right|\sin(\omega t+\psi_J)\sin(\omega t+\psi_\varphi) \tag{34}$$

- average values for a cycle:

$$\Delta F_{\varphi T}(r_m,\varphi_m)=\left|J_m\right|\Delta S_m\left|B_r(r_m,\varphi_m)\right|[\cos(\psi_J+\psi_r)+2\sin\psi_J\,\mathit{sin}\,\psi_r] \tag{35}$$

$$\Delta F_{rT}(r_m,\varphi_m)=\left|J_m\right|\Delta S_m\left|B_\varphi(r_m,\varphi_m)\right|[\cos(\psi_J+\psi_\varphi)+2\sin\psi_J\,\mathit{sin}\,\psi_\varphi] \tag{36}$$

The computation made with the use of the above relationships enable determining values of the forces and investigating the effect of changes in the busway geometric parameters on the values of active forces.

Maximal electrodynamical forces in the system occur under short-circuit and non-stationary conditions, when the transient component is superimposed on the stationary periodical one of the short-circuit current. In result, some other phenomena, apart from sinusoidal patterns, may appear. For purposes of developing the calculation method the knowledge of the total non-stationary pattern of short-circuit current as a time-function was assumed. The transformations are similarly performed. The calculation is made with the use of the Laplace transform and, afterwards, the time-dependent values are reinstated.

Determination of voltage gradients in the system

Distribution of electric field in the system may be determined from the equations [2,7]:

$$\mathbf{E} = -\ \text{grad}\ V \qquad \text{and} \qquad \nabla^2 V = 0 \tag{37}$$

In the analyzed case maximal electric stress is determined based on analytical relationships (due to geometry of the system) [2]. Maximal values of electric field intensity occur un the points shown in Fig. 3.

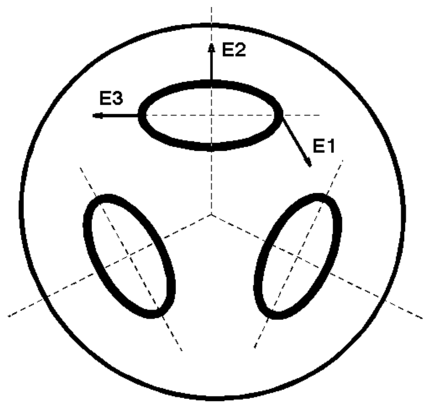

Fig. 3. The points of maximal values o electric field intensity

PROBLEMS OF HEAVY-CURRENT BUSWAYS OPTIMIZATION

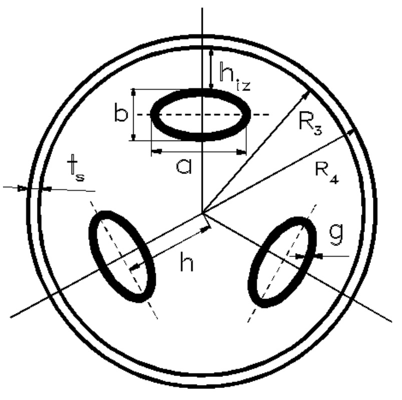
Fig. 4: The geometry of analyzed system

Geometric parameters characterizing the system (Fig. 4) are the following: thickness of the conductor wall (g), big (a) and small (b) oval diameters, the height of suspension of the conductors (h), and internal shield radius (R_3). For purposes of the present consideration a constant shield thickness (t_S) is assumed – amounting to 3 mm. This is imposed by technological requirements in manufacturing and exploitation of the busways. Outer shield radius (R_4) and insulator height (h_{iz}) are determined by the above dimensions. Thus, the system is characterized by five independent variables (a, b, g, h, R_3).

The optimization is aimed at finding the most favourable shape and geometric dimensions of considered busways in order to achieve: a) minimal consumption of structural material of conductors and shield (material optimization); b) minimal power losses (falling to unit length of the conductor) and consumption of structural material of the busway (power optimization – aimed at determining busway dimensions for possible the lowest busway exploitation cost, taking into account material consumption of the conductors and the shield). Hence, the optimization criterion includes, among others, a factor of variable costs of busway construction and exploitation.

The objective function S(u) depends on geometric variables affecting the value of busway cross-section area (the investment costs k_{inv}) and the level of active power losses (operation costs k_{op}) [8]:

$$S(\mathbf{u}) = f(u_1, u_2, ..., u_r) = k_{inv} + y \cdot k_{op} \tag{38}$$

where:

u – the vector of decisive variables (in the case of r=5, they are a, b, g, h, R_3),

k_{inv} – the investment cost, k_{op} – the exploitation cost.

Constraints of the optimization process are admissible electrodynamical and thermal parameters (temperatures of working lines T_{Cmax} and of the screen T_{Smax}, electrical strength of the system E_{max}, the forces acting in steady conditions and in short-circuit state F_{max}), as well as admissible ranges of variation of the geometrical dimensions.

A set of the above mentioned constraints Z_i in a general form is given by the function:

$$Z_i(\mathbf{u}) = [\ Z_1(\mathbf{u}), Z_2(\mathbf{u}), \dots , Z_k(\mathbf{u})\ ] \tag{39}$$

In order to convert the presented problem to a constraint-free optimization problem the criterion function $S_Z(\mathbf{u})$ has been modified [1,8]:

$$S_Z(\mathbf{u}) = S(\mathbf{u}) + \sum_{i=1}^{k} P_i\,(Z_i)^2\, N[Z_i] \tag{40}$$

where: $N[Z_i] = \begin{cases} 1 & \text{for} \quad Z_i > 0 \quad \text{infringement of the constraints} \\ 0 & \text{for} \quad Z_i \le 0 \quad \text{fulfilment of the constraints} \end{cases}$

The formula (40) includes P_i - a so-called penalty factor, of high value, being positive for the case of minimization $S(\mathbf{u})$ (and negative for maximization).

Due to the dependence of criterion function included extremized variables on delimiting parameters (the values of which are to be determined in result of complex numerical calculation) and due to the constraints included in the objective function, the modified criterion function can not be written in an explicit form. This imposes an additional problem that complicates the optimization calculations and considerably extends the time consumed for getting the results.

As an optimization method the genetic algorithms have been applied [1,5], as the most effective methods of optimum searching in a global sense. The selection was made based on the model of expected values and random selection with respect to remainders without repetitions, with the best individual "always" transmitted to the next generation. The population was of constant size, equal to 20 individuals. Crossing and mutation probabilities were constant and amounted to 0.8 and 0.005, respectively. The optimization process was performed for 100 generations.

RESULTS OF THE CALCULATION

The optimization calculation was effected for three-phase shielded busways, operating in symmetrical cylindrical arrangement (Fig. 4), made of aluminum, with air used as an insulating medium. For assumed rated voltage and current, material parameters and temperatures (of the environment, conductors, and shield) such optimal geometrical dimensions of the heavy-current busways were determined that lead to satisfaction of the selected optimization criterion (the material or energetic one) and ensuring keeping admissible electrodynamical parameters of the system.

For purposes of the calculation the following exploitation and material data have been assumed: rated voltage 10 kV, rated current 1÷4 kA, temperature of the environment 308 K, temperature of the shield 338 K, operation temperature of the conductors 363 K, conductivity of conductor and shield material $37{\cdot}10^6$ S/m, temperature coefficient of resistance variation of the conductors and shield 0.004 1/K, emissivity coefficients of outer surfaces of the conductors and shield amounting to 0.5; mains frequency 50 Hz.

Results of the optimization calculation for various current and voltage values are shown in Figs. 5 ÷ 8.

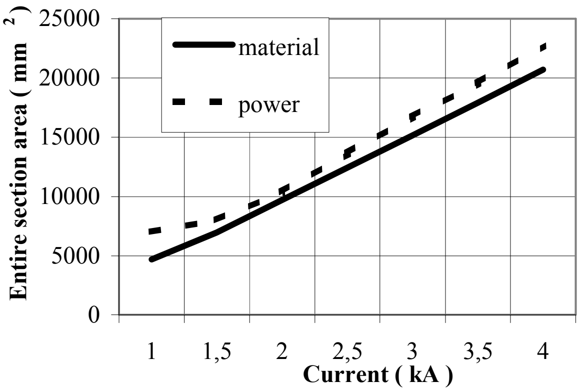

Fig 5. Total cross-section areas of heavy-current busways (obtained in result of material and power optimization) as the functions of varying rated current

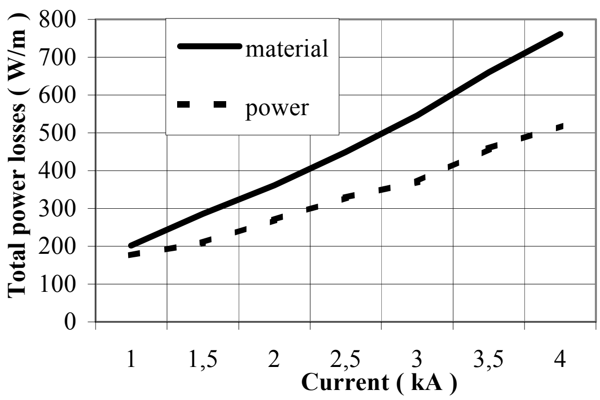

Fig. 6. Total active power losses falling to unit length of the busway (obtained in result of material and power optimization) as the functions of varying rated current

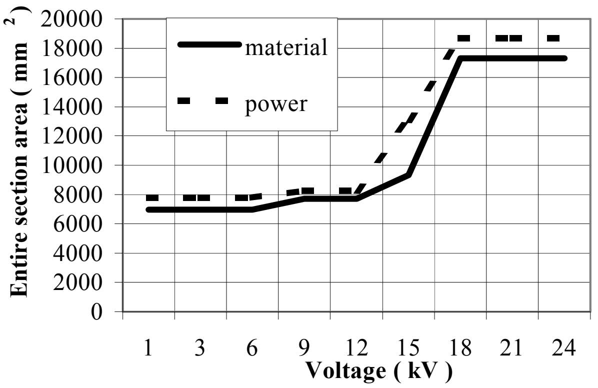

Fig 7. Total cross-section areas of heavy-current busways (obtained in result of material and power optimization) as the functions of varying rated voltage

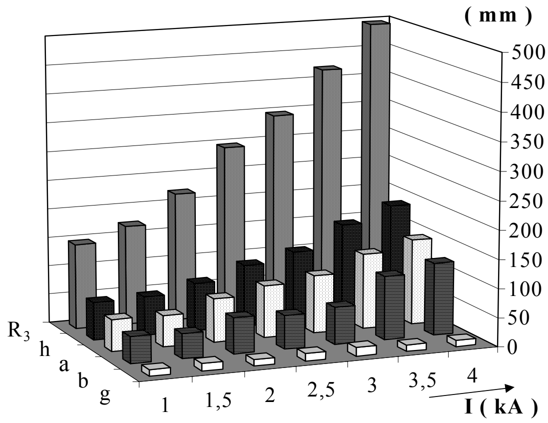

Fig. 8. The effect of changing rated current on optimal values of geometric dimensions of the busways, for $U_n = 3$ kV

FINAL REMARKS AND CONCLUSIONS

- Material optimization allows determining busway dimensions ensuring their minimal unit mass, maintaining their admissible electrodynamical and thermal conditions (leading to growing power losses in these structures). In result of power optimization such busway dimensions are obtained that ensure the lowest operational costs of the busways, taking into account the cost of conductor and shield material (with a view to achieving real geometric dimensions).
- In case of air-insulated heavy-current busways applied for middle voltages (up to 20kV) the optimization results depend chiefly on heat factors and to less degree on the other electrodynamical parameters. For low rated currents and higher rated voltages the optimization results are to higher degree affected by electric strength of the system.

- Temperature measurements of the conductors and shield performed in busway manufacturer's laboratories have been satisfactorily consistent with the calculations made with the use of the methods here referred to (the differences are under 1,5 per cent).
- The genetic algorithms are reckoned among the most effective optimization methods, enabling achieving the optimum in a global sense. Their specific properties enable easy paralleling of the calculation process, that results in considerable acceleration of the optimization process.
- More original effect of the performed calculation led to 30 per cent reduction of construction materials consumption that ensures reduction of busways mass and manufacturing cost.

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Bednarek K. Elektrodinamiskās un optimizācijas problēmas trīsfāzu stipro strāvu ovālas formas līnijām.
Darbā aprakstīta metode elektodinamisko parametru noteikšanai trīsfāzu stipro strāvu šinām un risinātas to šķērsgriezumu un formu optimizācijas problēmas .Tika izmantota integrālo vienādojumu metode, lai noteiktu strāvas blīvuma sadalījumu fāzu šinās kā arī strāvām, kuras inducējas ekranējošā daļā (izmantojot magnētisko vektoru potenciāla metodi iepriekš izvēlētās sistēmas daļās). Strāvu blīvuma sadalījums ļāva aprēķināt sistēmas aktīvos jaudas zudumus un temperatūru sadalījumu šinās un ekrānā. Izdevās aprēķināt sistēmas maksimālās elektriskās slodzes. Elektrodinamiskie aprēķini deva iespēju optimizēt stipro strāvu šinas. Kā optimizācijas kritēriji izmantoti izdevumi šinu izveidošanai un ekspluatācijai kā arī to izstrādāšanai. Optimizācijai lietots modificēts ģenētisks algoritms. Doti optimizācijas aprēķinu rezultāti un pētījuma rezultātu kopsavilkums.

K. Bednarek. Electrodynamical and optimiztion problems of oval three-phase heavy current lines.
The work presents a method determining electrodynamical parameters of three-phase heavy-current busways and the problems related to optimization of their shapes and cross-sections. The integral equations method served for determining the distribution of current density in phase conductors and the one induced in the shield (using the equations of magnetic vector potential for previously defined sub-areas of the system). The current density distribution allowed for calculating active power losses in the system and temperature distribution in the conductors and shield. Moreover, maximal electric stresses of the system were determined. The electrodynamical calculation enabled optimizing the geometry of the heavy-current busways. As the optimization criterion the investment and operation cost was chosen, related to design and operating the heavy-current busways. A modified genetic algorithm was applied for the optimization purpose. The results of optimization calculation are presented and the summary includes an analysis of the results obtained during the study.

Беднарек К. Проблемы электродинамики и оптимизации для трехфазных линий овальной формы для сильных токов.
В работе рассмотрен метод определения электродинамических параметров для трехфазных шин сильных токов а также решается проблема оптимизации сечения и формы этих шин. Для получения распределения плотности токов в шинах и в экране применен метод интегральных уравнений (для экрана использован метод векторных потенциалов для выбранных частей системы). Распределение плотностей тока позволяет определить потери активной мощности и температуру для шин и экрана. Определены максимальные мощности системы. Благодаря расчетам электродинамики, можно оптимизировать шины сильных токов. В качестве примеров оптимизации использованы расходы на изготовление, разработку и эксплуатации шин. Для оптимизации использован модифицированный генетический алгоритм. Приведены результаты оптимизации и дается обзор главных результатов разработки.