

INTERPOLATION FOR NON-REGULARLY LOCATED WELLS OF HYDROGEOLOGICAL MODELS

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1 INTRODUCTION

The vector φ of the piezometric head is the numerical solution of a boundary field problem which is approximated in nodes of a grid of a hydrogeological model (HM) by the following algebraic expression:

$$A \varphi = b, \quad A = A_{xy} + A_z - G, \quad b = \beta_\psi - G \psi \quad (1)$$

where the matrices A_{xy} , A_z , G represent, correspondingly, current transmittivity a_{xy} of aquifers (these links are arranged in xy -planes), the vertical ties a_z originated by aquitards (if the semi-3D scheme is used), the elements g_{xy} , g_z connecting nodes of the grid with the piezometric boundary conditions ψ , the vector β accounts for boundary flows. They also include the vector β_w of groundwater discharge/recharge from wells.

The φ and ψ -distribution of (1) must reproduce values of the head measured at monitoring wells. The matrix A must incorporate observed permeability and geometrical features of geological strata.

As a rule, locations of production and monitoring wells do not coincide with nodes of the HM grid. These locations may be represented as non-regular points that should be attached to the grid by interpolation. The roughest interpolation method moves these points to the one nearest node. This method not only worsens the accuracy of φ (due to shifting positions of production wells), but also deteriorates the role of monitored head values as calibration targets. These effects may be considerable for regional HM where the plane approximation step h is large (500 m – 4000 m).

This paper is devoted to interpolation for non-regular points of HM grid. The reported results represent recent development of methods described in (Lace et al., 1995). Interpolation for non – regular points is conditionally named as the forth and back one if it is used for forming HM and for transferring obtained results to these points, respectively.

2 FORTH INTERPOLATION FOR PRODUCTION WELLS

Forth interpolation for production wells is considered by using the scheme of Fig.1 for an elementary $h \times h$ block of a uniform grid. Within the block, a single flow source 0 is sited. Its flow β_0 should be interpolated among neighboring nodes $n = 5, 6, 7, 8$, as follows:

$$\beta_0 = \sum_{n=5}^8 \beta_0^n, \quad \beta_0^n = c_{0n} \beta_0, \quad \sum_{n=5}^8 c_{0n} = 1 \quad (2)$$

where the position of the source within the block depends on the local coordinates h_{0i} , $i = 1, 2, 3, 4$ (i - projections of 0 on edges), c_{0n} – the interpolation coefficients transferring β_0 to the nodes $n = 5, 6, 7, 8$.

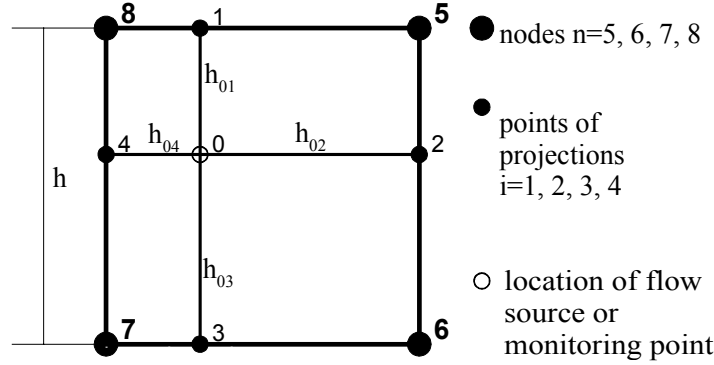


Fig. 1. An elementary $h \times h$ block with a flow source or a monitoring point

The following structure of c_{0n} results from two interpolation stages ($0 \rightarrow i$; $i \rightarrow n$) which performs elimination of β_0 :

$$\begin{aligned} c_{05} &= c_{01} c_{15} + c_{02} c_{25}, & c_{06} &= c_{02} c_{26} + c_{03} c_{36}, \\ c_{07} &= c_{03} c_{37} + c_{04} c_{47}, & c_{08} &= c_{04} c_{48} + c_{01} c_{18}. \end{aligned} \quad (3)$$

The intermediate coefficients c_{0i} and c_{in} represent the stages ($0 \rightarrow i$; $i \rightarrow n$), respectively. The projection coefficients c_{0i} are computed, by applying the inverse distance method (IDM), as follows:

$$c_{0i} = a_{0i} / a_{00}, \quad a_{00} = \sum_{i=1}^4 a_{0i}, \quad a_{0i} = \sigma_i h / (h_{0i} + \varepsilon h)^v, \quad i = 1, 2, 3, 4. \quad (4)$$

For the original version of (4), $v=1.0$ was used. Due to reasons explained later, $v=1.4$ provides better results. The current transmittivity σ_i at the point i depends both on its position on the edge and on the transmittivities σ_n at nodes ending the edge:

$$\begin{aligned} \sigma_1 &= ((h_{02} + \varepsilon h) / h \sigma_8 + (h_{04} + \varepsilon h) / h \sigma_5)^{-1}, \\ \sigma_2 &= ((h_{03} + \varepsilon h) / h \sigma_5 + (h_{01} + \varepsilon h) / h \sigma_6)^{-1}, \\ \sigma_3 &= ((h_{04} + \varepsilon h) / h \sigma_6 + (h_{02} + \varepsilon h) / h \sigma_7)^{-1}, \\ \sigma_4 &= ((h_{01} + \varepsilon h) / h \sigma_7 + (h_{03} + \varepsilon h) / h \sigma_8)^{-1}. \end{aligned} \quad (5)$$

In (4) and (5), the constant $\varepsilon \sim 10^{-5}$ averts the division by zero if $h_{0i} = 0$. Similar measures are needed for all interpolation formulas to be considered further. To simplify description of the formulas, their necessary ε -protection is not displayed. If $h_{0i} \rightarrow 0.5 h$, $\sigma_i \rightarrow a_{xy}$ of A_{xy} . The first version of (4) applied $\sigma_i = a_{xy}$ (Lace et al., 1995).

The coefficient c_{in} depends only on the position of the point i on the edge:

$$c_{15} = c_{36} = h_{04} / h, \quad c_{18} = c_{37} = h_{02} / h, \quad c_{25} = c_{48} = h_{03} / h, \quad c_{26} = c_{47} = h_{01} / h. \quad (6)$$

By introducing normalized distances $\rho_{0i} = h_{0i} / h$ and the local normalized coordinates ξ and η , with the node $n=7$ as the origin:

$$\xi = h_{04} / h = \rho_{04}, \quad 1 - \xi = h_{02} / h = \rho_{02}, \quad \eta = h_{03} / h = \rho_{03}, \quad 1 - \eta = h_{01} / h = \rho_{01} \quad (7)$$

and accounting for (6), the expression (3) takes the form:

$$\begin{aligned} c_{05} &= c_{01} \xi + c_{02} \eta, & c_{06} &= c_{03} \xi + c_{02} (1-\eta), \\ c_{07} &= c_{03} (1-\xi) + c_{04} (1-\eta), & c_{08} &= c_{01} (1-\xi) + c_{04} \eta. \end{aligned} \quad (8)$$

If in (4) $\nu=1.0$ and $\sigma = \text{const}$, the system (8) becomes much simpler:

$$c_{05} = \xi \eta, \quad c_{06} = \xi (1-\eta), \quad c_{07} = (1-\xi) (1-\eta), \quad c_{08} = (1-\xi) \eta. \quad (9)$$

The system of (9) represents the set of rectangular hyperbolas projected on the normalized block 1×1 . As an example, contours of $c_{07} = \text{const}$ of (9) are shown in Fig.2.a). These contours have the following features:

- forth interpolation of β_0 is linear on any line parallel to the edges of the block;
- $c_{07} = 0$ if $\xi = \eta = 1$ (edges 8 - 5 and 5 - 6); therefore, the influence region for the node $n = 7$ represents the 2×2 area containing four elementary blocks surrounding the node;
- if ξ or $\eta = 0$ (edges 6-7 and 7-8) then β_0 gets distributed between two nodes ending the edges;

However, the contours of Fig. 2a) are not circular with respect to the node $n = 7$, at its vicinity This drawback can be corrected if $\nu = 1.4$ is used for c_{0i} of (4). The improved contours are shown in Fig. 2b):

- their shape is still close to rectangular hyperbolas if $k_{07} < 0.35$;
- interpolation is linear on edges of the elementary block.

Not any forth interpolation method possesses useful features of (8). For example, the classic IDM gives:

$$c_{07} = \rho_{07}^{-1} / \sum_{n=5}^8 \rho_{0n}^{-1} \quad (10)$$

where ρ_{0n} are normalized distances r_{0n} / h In Fig.2c), the contours c_{07} given by (10) are shown. Their drawbacks are obvious:

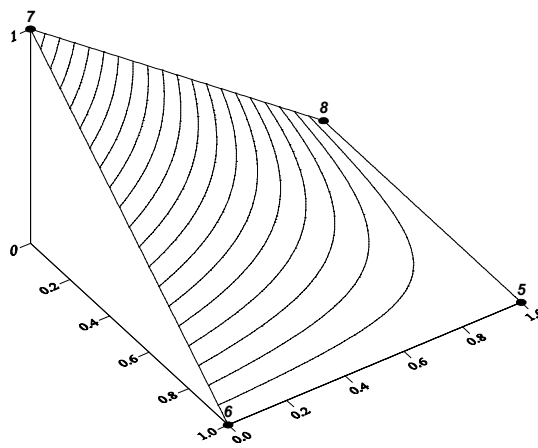
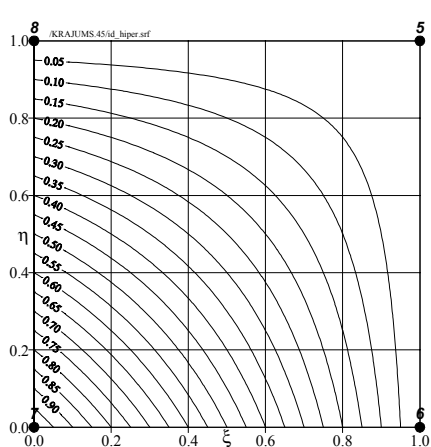
- interpolation is nonlinear in any direction;
- no borderline $c_{07} = 0$ exists; $c_{07} = 0$ only at nodes $n = 5, 6, 8$; due to this fact, it is not possible to set justly the limits for the area of influence of the node $n = 7$;
- if the source β_0 is located on an edge of the elementary block then not only the endpoints of the edge, but at least four neighboring nodes should be accounted for.

For a node n , the summary flow β_n resulting from forth interpolation of irregular β_j , which are located within the $2h \times 2n$ area of influence, is given by the formula:

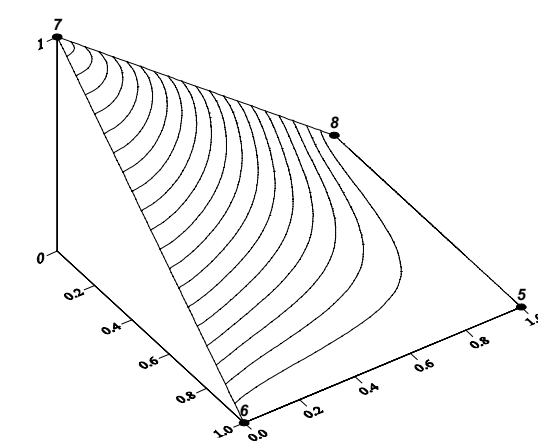
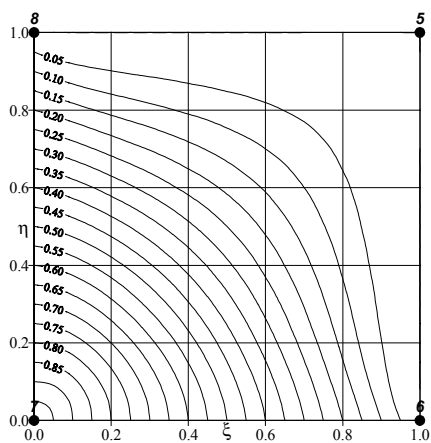
$$\beta_n = \sum_{j=1}^J \beta_j^n, \quad (11)$$

where β_j^n are the partial flows of β_j given by (2); J is the number of sources accounted for.

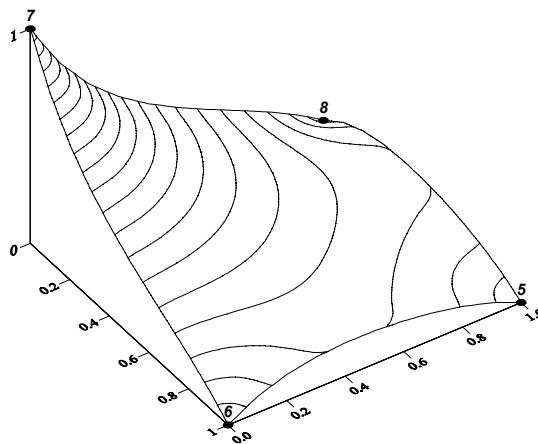
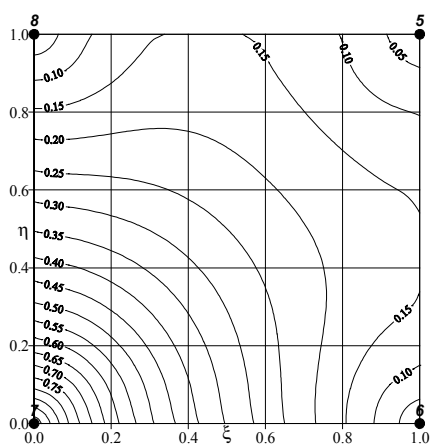
Forth interpolation of β_w improves accuracy of HM. Unfortunately, this advantage can be exploited only then if back interpolation is available for irregularly located production and monitoring wells. Formally, this difficulty does not exist for these modeling systems which roughly move these wells into the nearest node.



a) The set of rectangular hyperbolas obtained by (9)



b) Improved contours obtained by (8) if $\nu=1.4$



c) Contours obtained by the inverse distance method (10)

Fig. 2. Contours of c_{07} on the normalized grid block 1×1 ; $\sigma = \text{const}$

3 RESORATION OF HEADS FOR PRODUCTION WELLS

In comparison with forth interpolation of β_w , back interpolation for non-regular points is more complex. Recently, its original version (Lace et al., 1995) has been considerably improved and some new results are to be explained here.

Firstly, the value of φ_0 for the source of β_0 interpolated into the neighboring nodes must be restored. The following assumption is used:

$$\varphi_0 = \sum_{n=5}^8 \varphi_n c_{0n} + \tau_0 \beta_0 = \varphi_0^n + s_0, \quad (12)$$

where φ_n are the computed heads at four nodes of Fig.1; c_{0n} are the improved coefficients ($v=1.4$) of (8); τ_0 is the local hydraulic resistance for the source; φ_0^n , s_0 are the head and local depression, caused by the grid solution and the source, respectively. The value of τ_0 should be predicted for any location of β_0 within the elementary block. To simplify this task, it is assumed that $\sigma_n = 1$, temporarily.

For any node of the grid, $\tau_0 = 0$. The maximum of τ_0 is expected at the centre of the block where $c_{05} = c_{06} = c_{07} = c_{08} = 0.25$. The other characteristic locus is the middle of an edge where β_0 is distributed in equal parts between the two nodes ending the edge. These two special values of τ_0 were obtained experimentally, as described below.

The elementary block was conditionally placed at the central part of a homogenous grid ($\sigma = 1.0$) containing 100×100 nodes; on the borderline of the grid, the condition $\psi = 0$ was specified. A single movable unity source of $\beta_0 = 1.0$ was applied as the flow condition to be positioned and interpolated within the elementary block. Then the grid solution of (1) can be interpreted as the resistances τ_{int} at nodes with respect to the nullified borderline. The maximal possible value $\tau_m = 0.8874$ was obtained when the source was located exactly at the node. This value was practically constant for all nodes of the grid, but the ones located nearby the borderline. If the source was sited at the centre of the elementary block then the minimal value $\tau_{int} = 0.6842$ appeared at four nodes where the interpolated partial flows $\beta_0^n = 0.25$ were applied. The intermediate value $\tau_{int} = 0.7634$ was obtained for two nodes if the source was in the middle of an edge. The local resistance τ_0 to be found is $\tau_0 = \tau_m - \tau_{int}$. Results of the experiment are summarized in Table 1.

Table 1. Computed resistances for various positions of the unity source

Nr	Position of source	Resistance at node τ_{int}	Local resistance τ_0	Equivalent radius r_s of source
1	node	0.8874	0	0.1972 h
2	edge	0.7634	0.1240	0.4299 h
3	centre	0.6842	0.2032	0.7071 h

The following analytic formula which gives the resistance τ between two coaxial cylinders (R and r are, correspondingly, radii of the outer and inner cylinders):

$$\tau = (\ln(R/r)) / 2\pi\sigma \quad (13)$$

is applied for computing the equivalent radius r_s of the interpolated source. If $\tau = \tau_{int}$, $\sigma = 1$ then $r_s = R / \exp(2\pi \tau_{int})$, where $R = 52.059 h$ approximated the borderline of the grid area containing 100×100 nodes.

The values of τ_0 from Table 1 are exactly repeated by (13) if $R = r_s$, $r = 0.1972 h$. There the ratio R/r does not include h . Therefore, τ_0 depends only on the position of the source within the block $h \times h$.

It follows from Table 1 that any source located exactly in a node has the equivalent radius $r_s = 0.1972 h$. It may be assumed that a non – regularly located source also has $r_s = 0.1972 h$. As a rule, $r_s > r_w$ where r_w is the real radius of the well. Due to this reason, the summary resistance τ_{0w} of the source is, as follows:

$$\tau_{0w} = \tau_0 + \tau_w, \quad \tau_w = (\ln(0.1972/\rho_w)) / 2\pi\sigma_0, \quad \rho_w = r_w / h, \quad \rho_w \leq 0.1972, \quad (14)$$

$$\sigma_0 = (\sum_{i=1}^4 \sigma_i \rho_{0i}^{-1}) / \sum_{i=1}^4 \rho_{0i}^{-1}$$

where τ_w is obtained by (13) if $R = 0.1972$; $r = \rho_w$; the current transmissivity σ_0 is IDM interpolation on σ_i of (5).

The surface τ_0 is the main element enabling to restore heads at production wells by using (12). The initial version of the empiric formula for computing of τ_0 , within the normalized block, was as follows:

$$\tau_0 = (\sum_{i=1}^4 a_{0i} / (0.3444 + 0.4960 a_{0i} / a_i))^{-1}, \quad a_{0i} = \sigma_i \rho_{0i}^{-1} \quad (15)$$

where a_i were given by the expressions:

$$a_1 = \sigma_1 / \xi (1 - \xi), \quad a_2 = \sigma_2 / \eta (1 - \eta),$$

$$a_3 = \sigma_3 / \xi (1 - \xi), \quad a_4 = \sigma_4 / \eta (1 - \eta). \quad (16)$$

The formula (15) confirms the experimental values from Table 1 ($\sigma = 1.0$). In Fig.3a), the contours of τ_0 given by (15) are shown. Contours of (15) have two disadvantages:

- in the vicinity of nodes, the contours are not circular towards the nodes as their origins;
- on edges, as borders between neighboring blocks, the values of τ_0 may not coincide when $\sigma_n \neq \text{const}$.

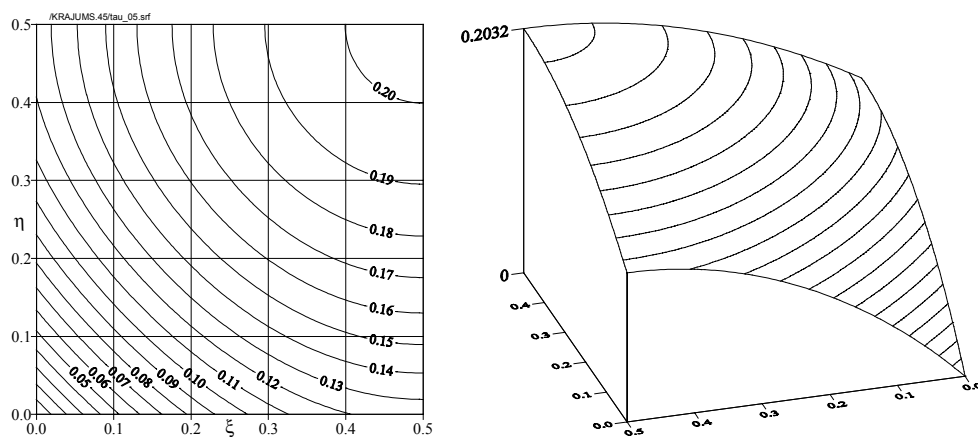
These drawbacks are eliminated in the following improved formula:

$$\tau_0 = (\sum_{i=1}^4 a_{0i} c_{0i} / (0.3444 + 0.5697 a_{0i} c_{0i} / a_i^{1.1}))^{-1}, \quad c_{0i} = \sigma_i \rho_{0i}^{-1.05} / \sum_{i=1}^4 \sigma_i \rho_{0i}^{-1.05}. \quad (17)$$

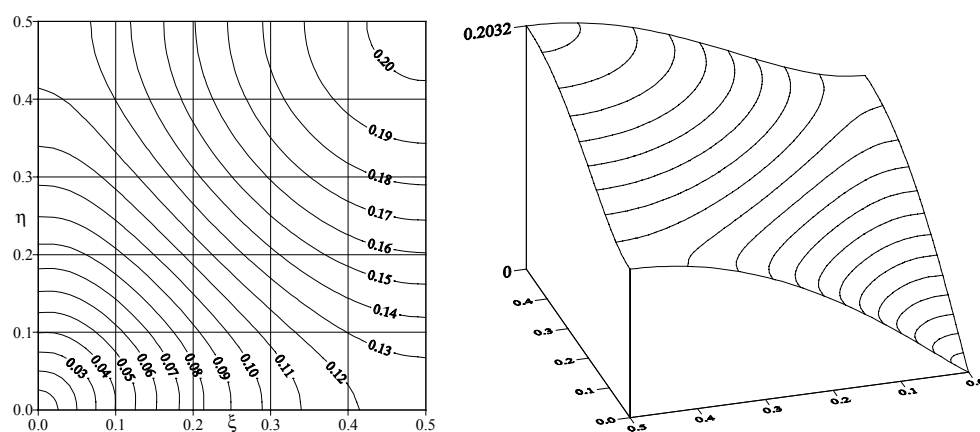
Due to introduction of c_{0i} , values of τ_0 for neighboring blocks coincide on edges bordering them. The elements a_{0i} , a_i are common for (15) and (17). By using the values 1.05 and 1.1 of powers, correspondingly, for c_{0i} and a_i , the circular shape of τ_0 was obtained in the vicinity of nodes.

In Fig.3b), the surface τ_0 of (17) is shown on the quarter of the normalized elementary grid block if $\sigma = 1$. In Fig. 3c), the graphs of τ_0 are the slices along the diagonal and the edge of the normalized block for the surface τ_0 .

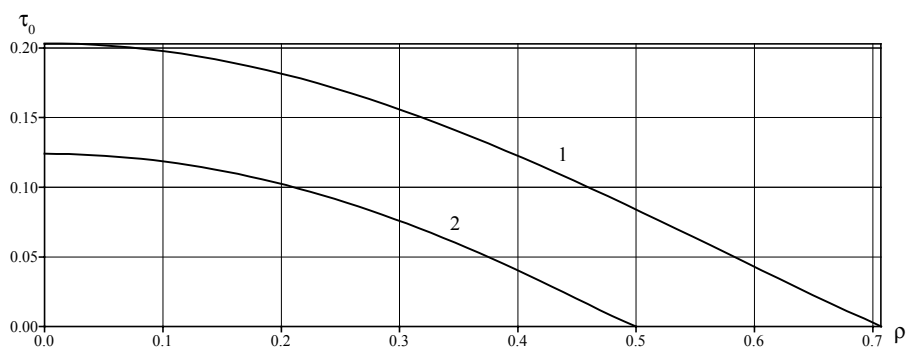
However, the formula (12) cannot give correct value of s_0 if other nearby located flow sources are present. To account for this situation, the full surface s_{0j} of the local depression cone caused by β_0 is necessary. This task is solved in the next section devoted to computing heads at monitoring wells.



a) Contours of τ_0 if (15) is used



b) Improved contours of τ_0 if (17) is used



c) Graphs of τ_0 slices along the diagonal (1) and the edge (2) for the block of Fig.3b)

Fig. 3. The surface of τ_0 on the quarter of the normalized grid block 1×1 ; $\sigma=1.0$

4. COMPUTING OF HEADS AT MONITORING WELLS

The task of computing the head φ_0 at the monitoring well of Fig.1 is more universal than the one devoted to restoring of the local maximum s_0 at the production well (formulas (12), (14)). It is assumed that the following expression should be used:

$$\varphi_0 = \sum_{n=5}^8 \varphi_n c_{0n} + s_{0j}, \quad s_{0j} = \tau_{0j} \beta_j \quad (18)$$

where τ_{0j} is the transfer resistance of the source β_j towards the monitoring point 0. If the distance $\rho_{0j} \rightarrow 0$ then $\tau_{0j} \rightarrow \tau_0$ and (18) \rightarrow (12).

The value of τ_{0j} must be zero at any node. The surface of τ_{0j} must be flat on the level τ_j (given by (17)) within a circle with the centre β_j and the radius $\rho = 0.1972$. Such a surface may be approximated by the modified IDM, as follows:

$$\tau_{0j} = \tau_j \rho_{0j}^{-3.0} / (\rho_{0j}^{-3.0} + \sum_{p=1}^P \rho_{0p}^{-1.5}), \quad \rho_{0p} \leq 2.0, \quad \rho_{0j} \leq 2.0 \quad (19)$$

where ρ_{0j} and ρ_{0p} are the normalized distances between the monitoring point 0 and the source β_j and the nearby nodes $p = 1, 2, \dots, P$ correspondingly; these distances should not exceed 2.0. The expression (19) is empiric. It was calibrated for the elementary block by considering the two characteristic source positions ($\sigma_n = 1.0$): 1) the center, 2) the middle of an edge. The results for the centre are shown in Fig.4 and Fig5. In Fig.4a), the contours of τ_{0j} are exposed. The top $\rho_{0j} \leq 0.1972$ of the surface τ_{0j} is not ideally flat on $\tau_{0j} = 0.2032$ and this fact causes errors. In (19), the powers -3.0 and -1.5 are chosen to minimize the error Δ_{0j} , along the diagonal of the normalized block 1×1 (Fig.5b):

$$\begin{aligned} \Delta_{0j} &= (\ln(1/\rho_{0j} \sqrt{2})) / 2 \pi - \tau_{0j}, \quad \text{if } 1/\sqrt{2} \geq \rho_{0j} \geq 0.1972, \\ \Delta_{0j} &= 0.2032 - \tau_{0j}, \quad \text{if } 0.1972 > \rho_{0j} > 0. \end{aligned} \quad (20)$$

where the analytic standard of (20) is represented by (13) if $1/\sqrt{2} \geq \rho_{0j} \geq 0.1972$, $R = 1/\sqrt{2}$.

It follows from Fig.5b) that the graph of Δ_{0j} has two maximal values 0.014 and -0.012 when $\rho_{0j} = 0.2$ and 0.35, respectively. Therefore, the relative error $100 \Delta_{0j} / 0.2032$ given by (20) does not exceed 7%.

If $\rho_{0j} < 0.1972$, the following analytic correction is necessary:

$$(\tau_{0j})_w = \tau_{0j} + (\tau_{0j})_{0.2h}, \quad (\tau_{0j})_{0.2h} = (\ln(0.1972/\rho_{0j})) / 2 \pi \sigma_j. \quad (21)$$

There $(\tau_{0j})_{0.2h}$ represents the analytic complement provided by (13) if $R = 0.1972h$. In Fig.4b), contours of $(\tau_{0j})_{0.2h}$ are shown if the minimal $\rho_{0j} = 0.25 \times 0.1972$. The contours of the summary surface $(\tau_{0j})_w$ are represented by Fig.4c). Due to the error Δ_{0j} caused by the slantwise top of τ_{0j} , the junction of the surfaces τ_{0j} and $(\tau_{0j})_{0.2h}$ is not smooth.

Set in Fig.6, the results are presented when β_j is set at the middle of the edge. Contours of τ_{0j} are shown in Fig.6a). For the source β_j , $\tau_{0j} = \tau_j = 0.124$. The error Δ_{0j} of τ_{0j} is evaluated on the edge where β_j is positioned:

$$\begin{aligned} \Delta_{0j} &= 0.837 (\ln(0.5/\rho_{0j})) / 2 \pi - \tau_{0j}, \quad \text{if } 0.5 \geq \rho_{0j} \geq 0.1972, \\ \Delta_{0j} &= 0.124 - \tau_{0j}, \quad \text{if } 0.1972 > \rho_{0j} > 0. \end{aligned} \quad (22)$$

It follows from the graph of Δ_{0j} provided by (22) that its maximum is $\Delta_{0j} = 0.0145$ when $\rho_{0j} \sim 0.2$ (Fig.6c). It is caused by the slantwise top of the surface τ_{0j} if $\rho_{0j} < 0.2$. This maximum is practically the same as for the graph of Fig.5b).

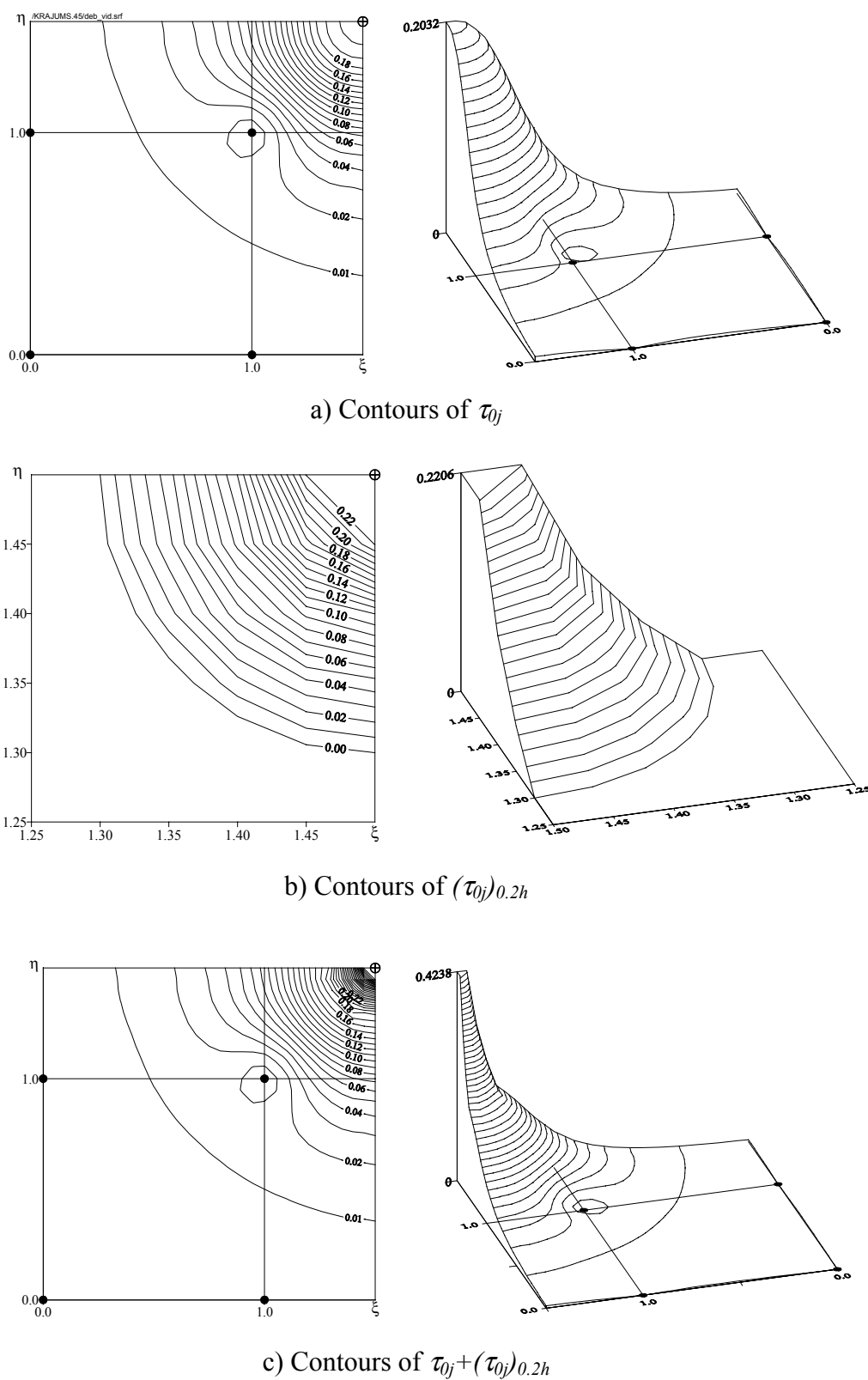
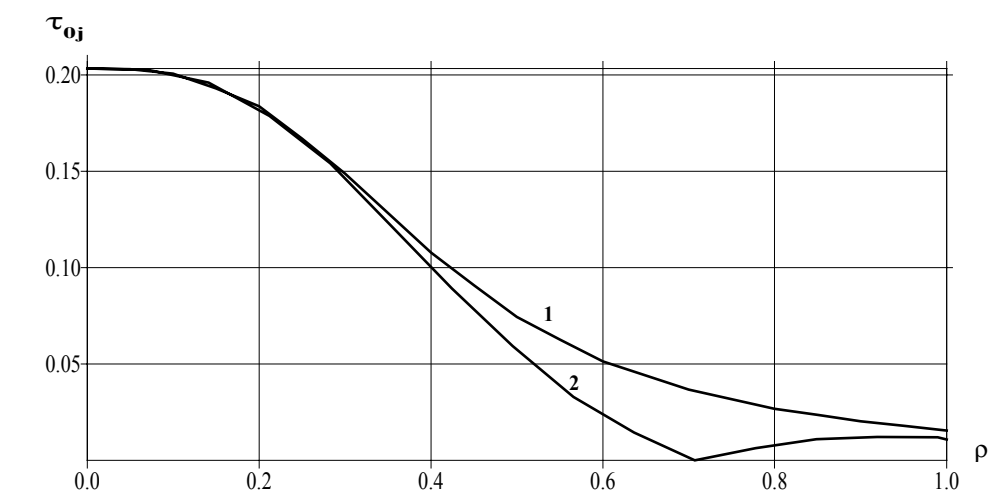
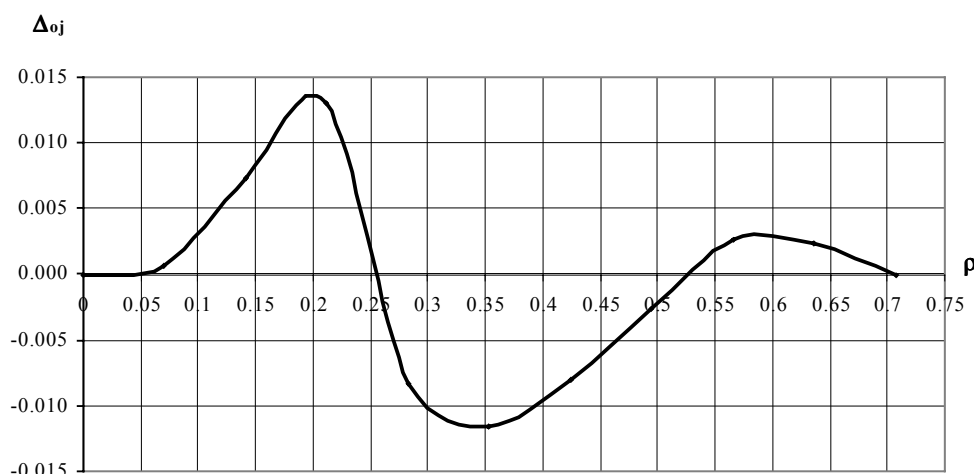


Fig. 4. The source at the center of the normalized block. Contours of τ_{0j} on the quarter of the region 3×3 ; $\sigma = 1.0$



a) Graphs of τ_{0j} for slices along the diagonal (1) and parallel to the edge (2)



b) Graphs of Δ_{0j} along the diagonal if $1/\sqrt{2} \geq \rho \geq 0$

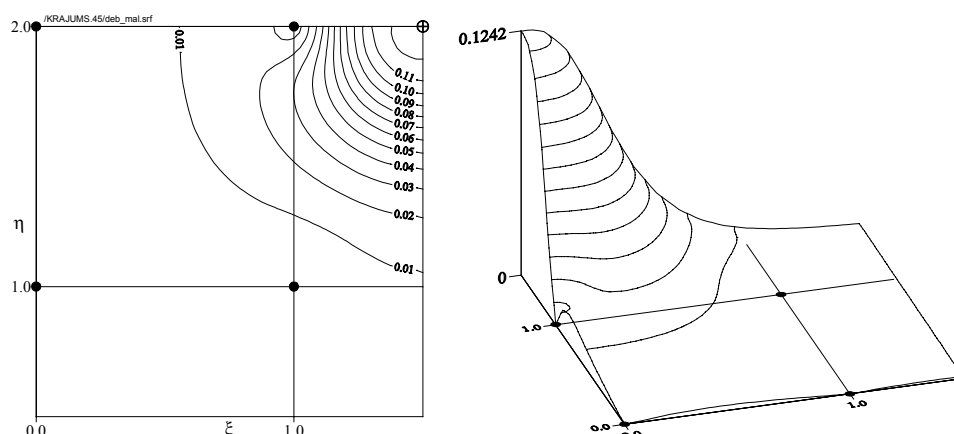
Fig. 5. Source at the center of the elementary block. Graphs of τ_{0j} and Δ_{0j} for slices through the source; $\sigma = 1.0$

If the surface τ_{0j} of (19) is used as a tool for computing of s_{0j} at the monitoring well 0 then it is possible to account for the influence of various sources β_j , $j = 1, 2, \dots, J$. They are located within a circle of the radius $\rho_{0max} = 2.0$. With the centre 0 where superposition of s_{0j} is applied and the final formula is generalization of (18):

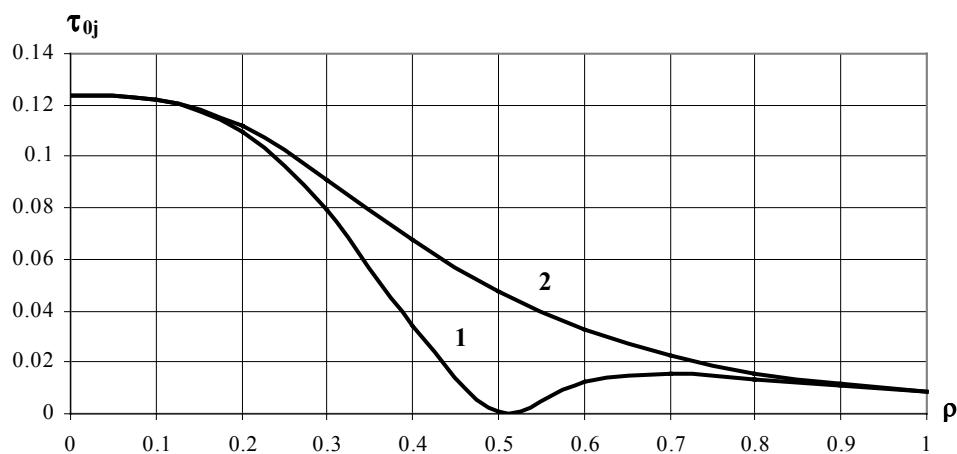
$$\varphi_0 = \sum_{n=5}^8 \varphi_n c_{0n} + \sum_{j=1}^J s_{0j}, \quad s_{0j} = \tau_{0j} \beta_j. \quad (23)$$

It is supposed that the nearest source is β_1 , $j = 1$, $\rho_{01} = \min$. If $\rho_{01} < 0.1972$ then the analytic complement of (21) should be used for obtaining of $s_{01} = (\tau_{01})_w \beta_1$. When $\rho_{01} \rightarrow \rho_{w1}$, then $s_{01} \rightarrow s_1 = \tau_{1w} \beta_1$ of (14), as the maximum of the local depression caused by β_1 .

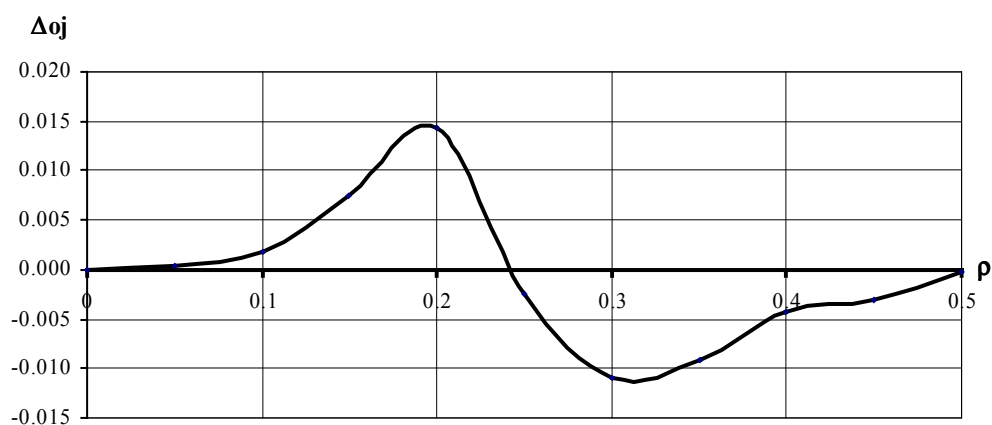
If compared with the original version of back interpolation, the expression (23) is more universal. Formulation of its main components τ_j and τ_{0j} have been improved considerably.



a) Contours of τ_{0j} on the quarter of the region 3×3



b) Graphs of τ_{0j} for slices through the source along the edge (1) and



c) Graphs of Δ_{0j} along the edge if $0.5 \geq \rho \geq 0$

Fig. 6. The source at the middle of the edge of the normalized block; $\sigma=1.0$

5 CONCLUSIONS

1. Methods for forth and back interpolation have been developed for non-regularly located production and monitoring wells.
2. The methods improve accuracy of hydrological models, especially, if grid plane steps are large.
3. The relative error of back interpolation does not exceed (5-10%)
4. The improved interpolation methods have been implemented practically.

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Spalviņš A., Šlangens J., Lāce I. Interpolācija neregulāri izvietotiem urbumiem hidrogeoloģiskajos modeļos.

Darba un novērošanas urbumu novietojums nesakrīt ar hidrogeoloģiskā (HM) mezgliem un šos novietojumus var uzskatīt par neregulāriem punktiem, kurus jāpiesaista HM ar interpolācijas palīdzību. Darbs ir veltīts šāda veida interpolācijai, kura uzlabo HM precizitāti. Tiek aprakstīta jauna metožu attīstība un rezultāti, kuri ir īpaši nozīmīgi reģionālajiem HM, jo to aproksimācijas režģu solis ir liels.

Spalvins A., Slangens J., Lace I. Interpolation for non-regularly located wells of hydrogeological models.

Locations of production and monitoring wells do not coincide with nodes of hydrogeological model and these locations may be considered as non – regular points that should be attached to HM by interpolation. The paper is devoted to this type of interpolation that improves accuracy of HM. New development and results are reported that are especially important for regional HM where approximation grids are coarse.

Спалвиньш А., Шланген Я., Лаце И. Интерполяция для нерегулярно расположенных скважин в гидрогеологических моделях.

Расположение рабочих и наблюдательных скважин не совпадает с узлами гидрогеологических моделей (ГМ). Эти скважины можно считать нерегулярными точками, которые следует включить в ГМ путем интерполяции. Работа посвящена такому виду интерполяции, которая повышает точность ГМ. Излагается новое развитие методов и результаты, которые особенно важны для региональных ГМ имеющих грубые сетки аппроксимации.