Arrangement of boundary conditions for hydrogeological model of Latvia

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***Abstract* – In 2012, scientists of Riga Technical University have established the regional hydrogeological model (HM) of Latvia (LAMO). LAMO covers 475km×300km area. It includes the territory of Latvia, the Gulf of Riga (active area of HM) and border areas of neighboring countries (presently, passive area of HM). The both areas are separated by the border zone where outer boundary conditions for the active area must be fixed. LAMO simulates the steady state regime of the active groundwater zone that contains drinkable groundwater resources for Latvia. LAMO includes 25 geological layers and HM plane approximation step is 500 metres. The active LAMO volume is enveloped by its top and bottom surfaces, but the outer vertical surface of border zone constitutes the shell of HM. For the HM top and bottom surfaces and for the border zone, piezometric boundary conditions are arranged. Presence of the HM passive part turns the shell surface impermeable for the transboundary groundwater flow. This distortion of the natural groundwater regime must be compensated by a proper choice of boundary conditions that are fixed within line of the border zone. On the LAMO top surface, the digital relief (terrain) map is used as the boundary condition. The map includes elements of the hydrographical network (rivers, lakes, sea). The rivers are attached to various geological layers, within the HM body. Special software has been developed to find the right attachment to HM over 200 larger rivers of Latvia. Lakes and sea are joined with HM through the aeration zone. The zone represents a formal aquitard that takes part in computing of infiltration flow for the HM. Because the digital relief is used as the boundary condition, HM itself provides feasible infiltration flow distribution that can be adjusted by its calibration. Due to original methods of establishing boundary conditions, LAMO has been created during a short time (2 years).**

***Keywords* –hydrogeological model, boundary conditions, infiltration flow, calibration of models**

I. INTRODUCTION

The countries of the world and of the European Union are developing hydrogeological models (HM) where, by means of computer modelling, the information necessary for the groundwater management planning is obtained. In 2012, scientists of Riga Technical University have established the regional HM of Latvia (LAMO). More detailed information regarding LAMO is given in publications [1, 2, 3, 4]. The present publication is focused on arrangement of boundary conditions for LAMO. Due to original methods used for establishing the boundary conditions, it was possible to develop LAMO during a short time (2 years).

LAMO covers 475km×300km area (Fig.1). It includes Latvia and border territories of neighbouring countries. For the current LAMO version, only Latvia and the Gulf of Riga constitute the HM active area (Fig.2), because, at present, no agreement exists regarding cross border modelling. The active and passive areas are separated by the 4km wide border zone that are used for fixing boundary conditions for the outside of the active area.

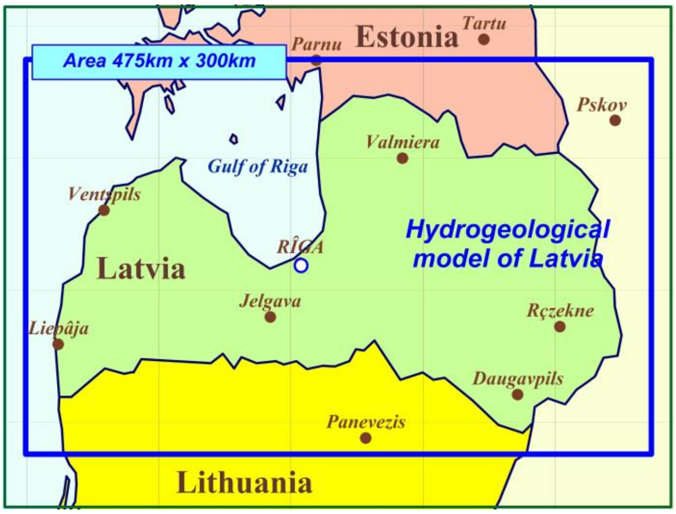

Fig. 1: Location of Latvia HM

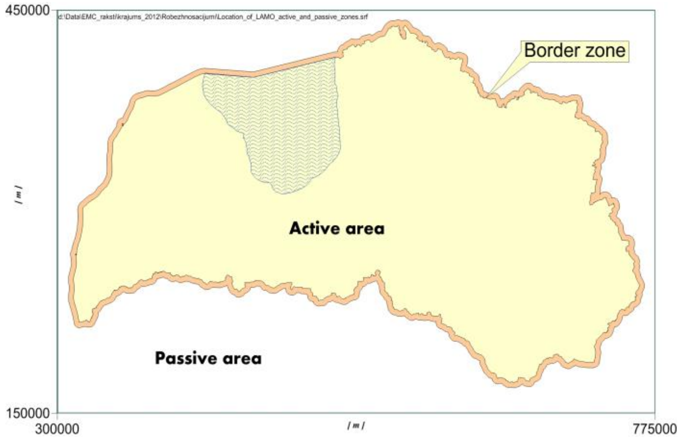

Fig. 2. Location of LAMO active and passive areas

To consider problems of arranging boundary conditions for LAMO, the basic mathematics of the 3D steady state model must be introduced. By applying the 3D finite difference approximation, the x, y, z – grid of HM is built using ($h \times h \times m$) sized bocks (h is block plane size, m is the variable thickness of a geological layer). The model constitutes a rectangular

p–tiered xy–layer system, where p is the number of geological layers. For LAMO, p=25, h=500 metres.

The modelling program controls the whole area of HM that contains also the passive area that takes no part in simulation. The active HM volume is enveloped by the border zone. Its outer vertical surface represents an impermeable shell that blocks transboundary groundwater flow. The relief (ground surface) and the lower side of the model are its geometrical top and bottom, accordingly. The 3D–space volume enveloped by the boundary surfaces (top, bottom, shell) constitutes the active body of HM. The vector φ of the piezometric head is the numerical solution of the boundary field problem which is approximated in nodes of HM xyz–grid by the following algebraic expression:

$$A\varphi=\beta-G\psi, \qquad A=A_{xy}+A_z \qquad (1)$$

where A is the symmetric sparse matrix of the geological environment which is presented by the xy-layer system containing horizontal (A_{xy} - transmissivity) and vertical (A_z – vertical hydraulic conductivity) elements of the HM grid; ψ - the boundary head vector: ψ_{rel}, ψ_{bot}, ψ_{out} ψ_{riv} - subvectors of ψ that represent boundary conditions the HM top, bottom, border zone, rivers, accordingly; G – the diagonal matrix (part of A) assembled by elements, linking the nodes where φ must be found with the points where ψ is given (for ψ_{top}, ψ_{bot}, ψ_{out} these points are nodes of the HM grid); β - the boundary flow vector.

The elements a_{xy}, a_z of A_{xy}, A_z (or g_{xy}, g_z of G) are computed, as follows:

$$a_{xy}=k\times m,\ a_z=\frac{h^2\times k}{m}$$
$$m_i=z_{i-1}-z_i>0,\ i=1,2,\ldots,p \qquad (2)$$

where z_{i-1}, z_i are elevations, accordingly, of the top and bottom surfaces of the i-th geological layer; z_0 represents the ground surface elevation ψ_{rel}-map with the hydrographical network included; k, m are, respectively, elements of digital m, k-maps of the computed layer thickness and permeability. If in some areas m_i=0 , then the i-the layer is discontinuous. To prevent the "division by zero", in the a_z calculation of (2), m_i=0 must be replaced by small $\varepsilon > 0$ (for LAMO, ε=0.02 metres).

For LAMO the most important are the boundary conditions ψ_{rel}, ψ_{out} ψ_{riv} . The condition ψ_{bot} gives small impact on HM, because the thick regional aquitard of D2nrz practically blocks vertical groundwater flow through the LAMO bottom surface. In LAMO, the β–vector is represented only by the withdrawal rates of well fields. It is not applied for usual simulation for recharge and evaporation flows on the HM top.

For LAMO outer surfaces, the ψ –conditions were applied, instead of the β–flows due to the two main reasons:

- the ψ–conditions shortened time needed for solving of (1), because they caused appearance of the matrix D as the diagonal dominance factor of A; [5, 6];
- numerical values of the ψ–conditions were known much better than the ones of the β–flows.

II. THE DIGITAL RELIEF MAP AND THE HEAD DISTRIBUTION OF THE D2pr AQUIFER

Appliance of piezometric boundary condition ψ_{rel} (Fig. 3), on the HM top instead of the conventional recharge and discharge (evaporation) flows (β_{inf} –conditions for infiltration flow) has considerably reduced the effort of establishing HM, especially, of the large regional models where the groundwater infiltration distribution is very complex. It must account for numerous recharge and discharge areas. No modeller is able to guesswork the right β_{inf} distribution for large 3D HM.

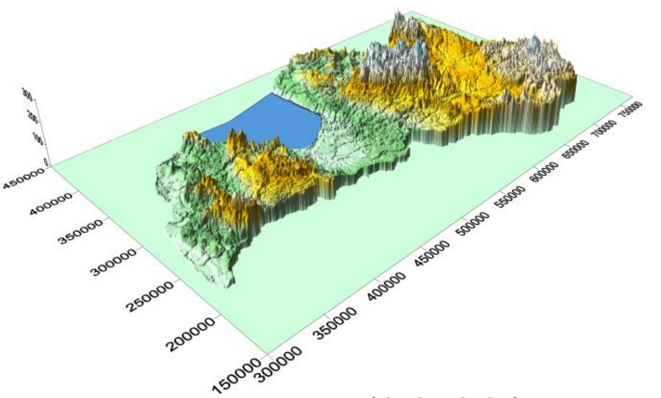

Fig. 3. Isometric image of the digital relief

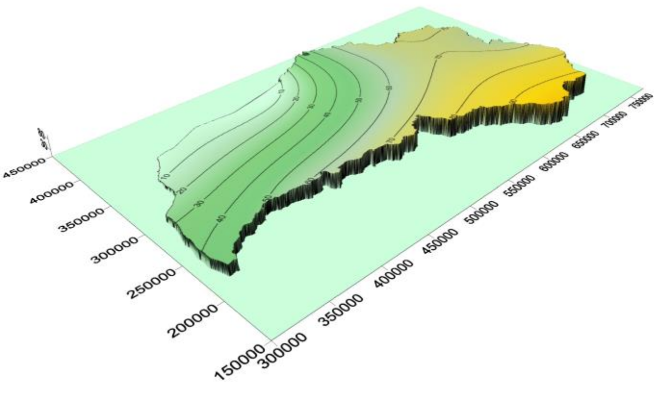

Fig. 4. Isometric image of the D2pr head distribution

The ψ_{rel}-map was obtained by using data of the Geospatial Information Agency of Latvia. Methods used to create the map are described in [7].

If the ψ_{rel} – map is used, the flow $q_{inf}=q_{aer}$ passes through the aeration (vadose) zone:

$$q_{aer}=G_{aer}(\psi_{rel}-\varphi_{Q2}) \qquad (3)$$

where φ_{Q2} is the computed head (subvector of φ) for the aquifer Q_2; G_{aer} (diagonal submatrix of G) contains the vertical links g_{aer} of the aeration zone connecting the fixed ψ_{rel} with the computed φ_{Q2}. The expression (3) gives the ordinary result of HM, when a ψ-condition is applied. As a rule, even the first run of HM provides feasible results for q_{inf}.

The vertical links, g_{aer} of the diagonal matrix G_{aer}, are controlling the q_{aer} distribution. Values g_{aer} depend on h^2, k_{aer} and m_{aer} (formula (2)) where h=500, k_{aer} and m_{aer} are, accordingly, the permeability and thickness of the aeration zone. Initially, k_{aer} and m_{aer} are unknown. In nature, $m_{aer}=\varphi_{rel}-\varphi_{Q2}$ if $q_{aer}>0$. If $q_{aer}<0$ then m_{aer} ceases to exist, in the form when $q_{aer}>0$, because the negative infiltration flow is caused mainly by lowlands, rivers and lakes. First, the following values of the aeration zone were tried: m_{aer}=0.02 [m], $k_{aer}=10^{-6}$ [m/day] (for recharge areas); $k_{aer}=10^{-4}$ [m/day] (for areas of lakes and sea); $k_{aer}=10^{-8}$ [m/day] (for areas of swamps). To avoid iterative changes of the HM geometry, m_{aer}=0.02 was kept constant, until the calibrated state of q_{aer}–flow was achieved by adjusting the k_{aer} – distribution.

The piezometric head distribution ψ_{D2pr} of the D2pr aquifer (Fig. 4) is applied as the boundary condition on the LAMO bottom surface. The ψ_{D2pr}-map was obtained by using information of [8] about the factors that formed the head distribution for the D2pr aquifer. Influence of ψ_{D2pr} is small, because it is separated from the HM body by the thick regional D2nr aquitard. There are two reasons for appliance of this boundary condition: it may be useful if a modeller carries out research regarding role of tectonic faults of the D2nr aquitard; the D2pr aquifer contains drinkable water at the North-East part of Latvia.

III. DESIGN OF BOUNDARY CONDITIONS FOR THE BORDER ZONE

Due to presence of the HM passive area, the outer vertical surface of the boundary zone (shell) is impermeable for the transboundary groundwater flow. This factor distorts the natural groundwater regime in the close vicinity of the shell. Because the border zone width is 4000 metres, the distortion is considerably smaller on the inner surface of the zone. There the flow, passing through this surface, exists. One must try to recover the natural groundwater regime on this border surface. It can be done by fixing auxiliary piezometrioc boundary conditions $\boldsymbol{\psi_{out}}$ on the middle line of the border zone. As the initial data for obtaining the $\boldsymbol{\psi_{out}}$ for aquifers of HM, three data sources were used that provided linewise information: 1. the $\boldsymbol{\psi_{rel}}$ –map; 2. $\boldsymbol{\psi_{out}}$ for the D2ar aquifer; 3. data extracted from the head distribution map of the prequaternary surface φ_{preQ}. The φ_{preQ} –surface can provide only fragments of $\boldsymbol{\psi_{out}}$ that can be observed from the bird's eye view. A special software was developed for extracting these data and for providing information that was necessary to design the interpolation tool that provided $\boldsymbol{\psi_{out}}$ for all aquifers of HM. Principles used to create the tool were reported in [9].

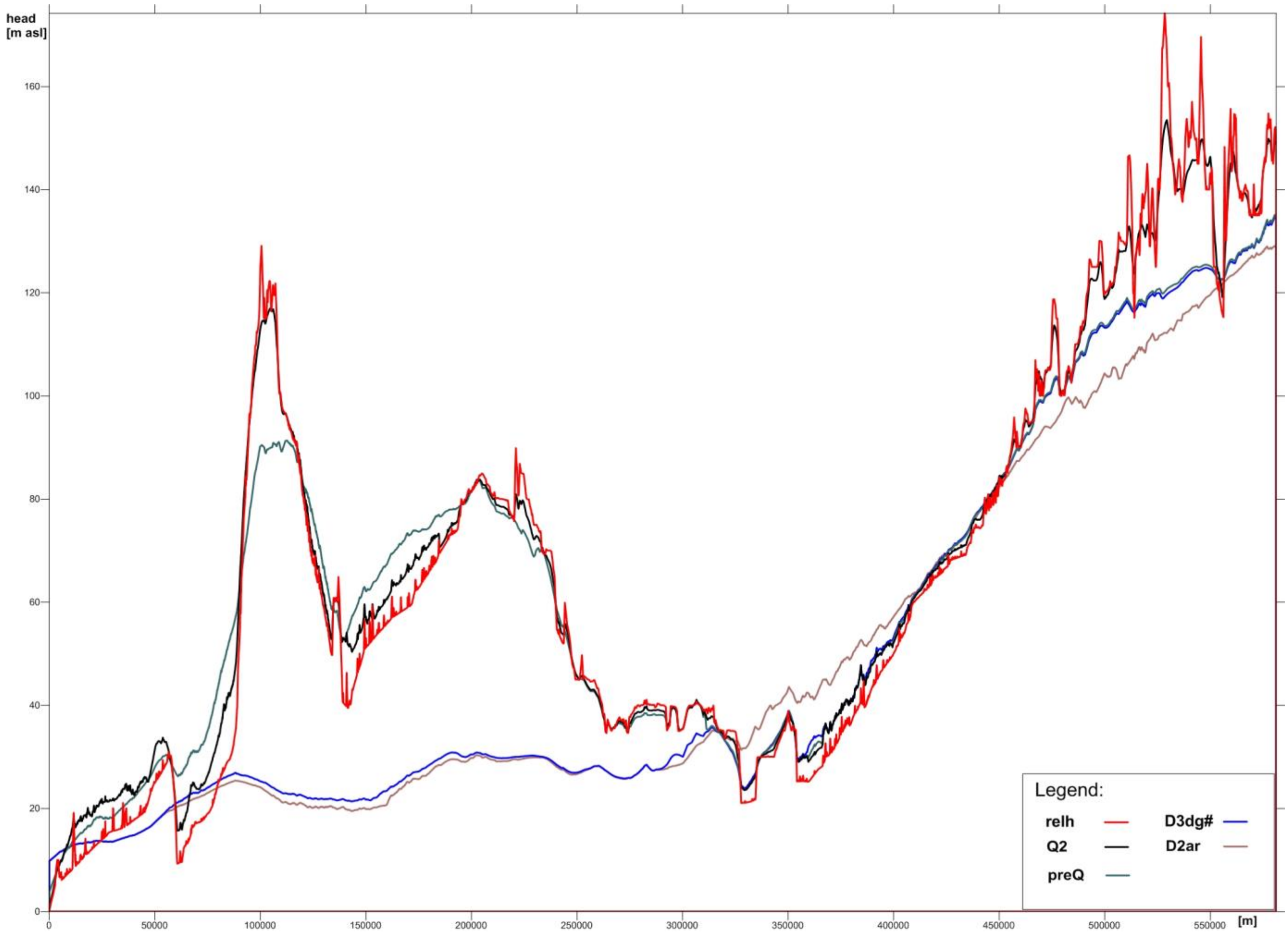

Fig. 5.

The design of the interpolation tool is based on the fact that, along the middle line of the border zone, a set of vertical links a_z exists that join neighbouring aquifers. These links account both for *k* and *m* parametres of the aquitards (formula (2)). If values of these links are considerably enlarged (at least 100 times) then this set of transformed vertical links behaves like a spatial interpolation device. It automatically provides the unknown components of $\boldsymbol{\psi_{out}}$.

The above mentioned software detected these locations of aquitards where they exist (m≠0). For the *m*=0 areas, no transformation of the a_z –values was done.

Therefore, the $\boldsymbol{\psi_{out}}$ boundary condition is obtained by HM itself, as follows:

- the components originated from the $\boldsymbol{\psi_{rel}}$, φ_{D2ar}, φ_{preQ}–maps are fixed; as the initial ones;
- the other components of $\boldsymbol{\psi_{out}}$ are supported by the spatial interpolation tool that contains transformed a_z–links.

Because the interpolator is not a part of the LAMO active volume, it can be used permanently. This feature is very useful during the HM calibration when the a_z –set gets changed and the ψ_{out} conditions follow these changes

IV. RIVERS AS BOUNDARY CONDITIONS

The considered above boundary conditions ψ_{rel} , ψ_{D2pr} , ψ_{out} are fixed on the outer surfaces of HM and they exist in nodes of the HM grid. The condition ψ_{riv} that represents water levels of rivers is attached to inner nodes of the HM grid via the set of the river bed conductances $G_{riv.}$ The flow q_{riv} caused by rivers is given by the matrix expression:

$$q_{riv} = G_{riv} (\varphi_{riv} - \psi_{riv}) \qquad (4)$$

where the diagonal matrix G_{riv} is a part *G*; φ_{riv} is subvector of φ, φ_{riv} represents these nodes where the corresponding components of ψ_{riv} are attached via the links belonging to G_{riv}. The vector ψ_{riv} is the subvector of ψ, but components of ψ_{riv} belong to a set of points located outside the HM grid. The value of a single element g_{riv} of G_{riv} is presented by the formula:

$$g_{riv} = h\, w_{riv}\, k_{riv} / m_{riv} \qquad (5)$$

were *h*=500 (plane step); w_{riv} – the width of river that is known and it is applied for computing g_{riv}; k_{riv}, m_{riv} – the permeability and thickness of the river bed layer, accordingly; these parametres are unknown. The value m_{riv}=1 was fixed and k_{riv}=0.002 was found experimentally. Therefore, as the initial try, the formula (5) gives g_{riv}=w_{riv}. For LAMO, rivers are presented by their middle lines. The only exception is the three artificial lakes of the Daugava river that are caused by the Riga, Kegums, Plavinas hydroelectrical power stations.

In Fig. 5, the set of rivers and lakes included in LAMO is shown. To obtain the ψ_{rel} –conditions, the following items must be prepared:

- the xy –location of a river line;
- the long line profile (water levels of a river along its line);
- the width of a river along its line;
- the z–attachment of a river line.

The last item accounts for the fact that a river, on its run, may be joined with different geological layers. For example (Fig. 6), the river Gauja runs through the Quaternary, the lower and upper Devonian layers. Hence, about 200 rivers of Latvia are included in LAMO, no modeller is able to join them

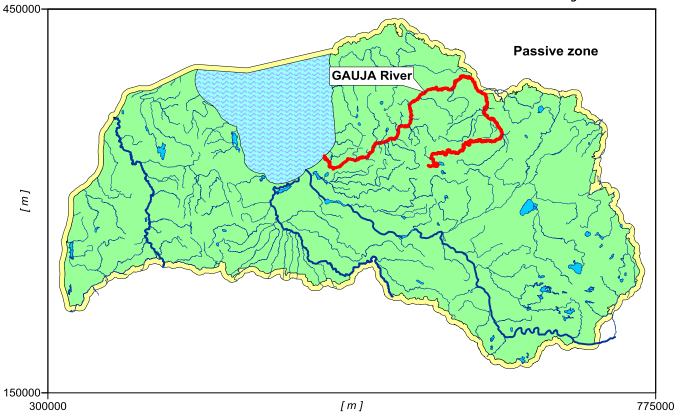

Fig. 6. Set of rivers and lakes included in HM

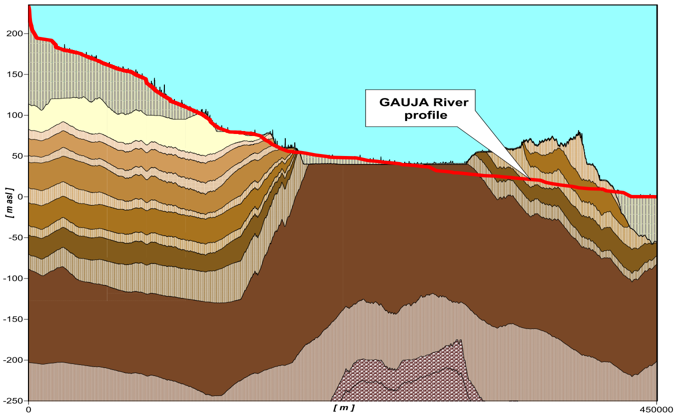

Fig. 7. Vertical cross section along the Gauja river

properly with a model. It was necessary to develop special software for preparing all data files that are needed for creating the ψ_{riv} – conditions. The program also performed the search for the river z –attachment. Presently, the empirical value k_{riv} =0.002 is applied for all rivers. In nature, k_{riv} may be different no only for each river, but also for their fragments. The task of finding more realistic k_{riv} distributions is very complex, because observed in nature river flows of (4) must be used. Knowledge of these flows provides more exact estimates of the interaction between groundwater and rivers.

V. CONCLUSIONS

Scientists of Riga Technical University have established the regional hydrogeological model of Latvia. Innovative methods used for arrangement of boundary conditions of this model enabled to shorten time and to ease efforts needed for creating this complex software tool. Appliance of the ground surface (terrain) map, as the boundary condition on the model top, enabled to obtain the infiltration flow distribution (the most important parameter) automatically, as the result provided by the model itself. Boundary conditions on the model outer vertical surface were also created by the model by transforming its outer surface into the interpolation tool that supported this boundary condition. Boundary conditions for rivers enabled to control river flows. Further efforts are needed to use this feature for more exact estimation of interaction between groundwater and rivers.

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