GEOLOGICAL PROFILES AS EFFICIENT MEANS FOR EXPOUNDING RESULTS PROVIDED BY HYDROGEOLOGICAL MODEL OF LATVIA

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ABSTRACT
In 2010-2012, scientists of Riga Technical University (RTU) have developed the hydrogeological model (HM) of Latvia (LAMO). LAMO represents the active groundwater zone of Latvia, as the source for providing drinking water. LAMO covers the area of 475km×300km and includes 27 geological layers; the plane approximation step is 500 meters. In 2013, the first practically important results have been provided by LAMO. The set of hydrogeological maps has been prepared to update the current water management plans for Latvia. For aquifers, maps of groundwater head and infiltration flow distributions have been prepared. Numerous maps for geological profiles have been obtained, because they are the most informative ones. To obtain correct maps for the profiles, it was necessary to develop a new interpolation method accounting for different comportment of the isolines within aquifers and aquitards of LAMO. Customary methods of making the isoline distributions for profiles result in wrong designs due to ignorance of this unlikeness. Theory of the proposed interpolation method is considered and results of its practical appliance are reviewed.

Keywords: hydrogeological model, hydrogeological maps, geological profiles, interpolation of data

1. INTRODUCTION
To obtain the information necessary for groundwater management, the countries of the world and European Union (EU) are developing hydrogeological models (HM). Latvia also is implementing aims laid down by the EU Water Framework Directive [1]. For the water management planning of a country, the directive proposes to apply the river catchment area principle. The territory of Latvia comprises four cross-border type river basin districts: those of Gauja, Daugava, Lielupe and Venta rivers (see Fig. 1).

The Latvian Environment, Geology and Meteorology Centre (LEGMC) has drawn down and adjusted the water management plans for these districts.

In 2012, the HM of Latvia (LAMO) has been established [2]. LAMO generalizes geological and hydrogeological information accumulated by LEGMC. The program GROUNDWATER VISTAS (GV) is used for running LAMO [3]. The GV program includes the MODFLOW system that is applied worldwide to support HM. The graphical program SURFER is applied for mapping the numerical simulation results [4].

In 2013, the first results for the Gauja river basin district [5] have been provided by the early LAMO version. After upgrading of LAMO, its improved results have been used to update the current water management plans for the river basin districts of Latvia.
In the paper, the recent materials on the Gauja river basin district [6] will be compared with the first ones taken from the report [5]. Numerous maps for geological profiles have been prepared. In Fig. 1, locations of regional profiles are shown. To create correct maps for the profiles, it was necessary to develop the new interpolation method.

2. MATHEMATICAL FORMULATIONS

To consider the problem of representation of the HM results, the basic mathematical formulations of the 3D steady state model must be introduced. By applying the 3D finite difference approximation, the xyz-grid of HM is built using \((h \times h \times m)\)-sized blocks; \(h=500\) metres is the LAMO block plane size; \(m\) is the variable thickness of a geological layer. The model constitutes a rectangular \(p\)-tiered \(xy\)-layer system. HM vertical sides compose the shell of the HM grid. Presently, only the territory of Latvia represents the active part of HM [2] and its shell coincides with the border zone that separates the HM active and passive parts. However, the GV system accounts for the whole body of LAMO (xyz-grid of the size 951×601×27).

Vector \(\varphi\) of the piezometric head is the numerical solution of the boundary field problem which is approximated in nodes of the HM grid by the following algebraic expression:

\[
A \varphi = \beta^{-1} \psi, \quad A = A_{xy} + A_z
\]  

(1)

where \(A\) is the matrix of the geological environment which is presented by the \(xy\)-layer system containing horizontal \((A_{xy}\) - transmissivity) and vertical \((A_z\) - vertical hydraulic conductivity) elements of the HM grid; \(\psi\) –the boundary head vector; \(G\) –the diagonal matrix (part of \(A\)) assembled by elements, linking the nodes where \(\varphi\) must be found with
the ones where $\psi$ is given; $\beta$ -the boundary flow vector that in LAMO is presented by discharges of water supply wells.

The elements $a_{xy}, a_z$ of $A_{xy}, A_z$ (or $g_{xy}, g_z$ of $G$) are computed as follows:

$$a_{xy} = k_i m_i ; \quad a_z = 2 h^2 k_i / (m_i + m_{i+1} k_i / k_{i+1}) ; \quad m_i = z_{i+1} - z_i > 0 \quad i = 1, 2, \ldots, p$$

where $z_{i+1}, z_i$ are elevations, accordingly, of the top and bottom surfaces of the $i$-th geological layer; $z_0$ represents the ground surface elevation $\psi_{rel}$ - map; $m_i, k_i$ are elements of digital $m_i, k_i$ - maps of the thickness and permeability of the $i$-th layer, accordingly; $m_{i+1}, k_{i+1}$ are parameters of the next underlaying grid plane of HM. The set of $z$-maps describes the full geometry (stratification) of LAMO. The set contains $p+1$ surfaces. The thickness $m_i$ of the $i$-th layer must be positive. If in some areas $m_i=0$ then the layer is discontinuous and $m_i=\varepsilon > 0$ must used (for LAMO, $\varepsilon = 0.02$ metres).

For the $i$-th layer, the nodes of the HM grid are located on the surface $z_{i,0.5}$ where the grid block centres are sited:

$$z_{i,0.5} = 0.5 (z_{i+1} + z_i), \quad i = 1, 2, \ldots, p.$$  

The components of the subvector $\phi_i$ are related to the surface $z_{i,0.5}$. They are stored as the grid file that is ready for visualization by the SURFER system. As examples of the SURFER products, for the $xy$ -type data, two maps: Fig. 2 and Fig 3 (taken from report [6]) are shown for the prequaternary surface preQ of LAMO.

![Fig. 2. Distribution of groundwater head [metres asl] for primary aquifers](image)

If one assumes that the $i$-th and $(i+1)$-th layers are an aquitard and the aquifer then the infiltration vector flow $q_{i,i+1} [\text{mm/year}]$ for the $(i+1)$-th aquifer can be computed by using the formula that has been introduced in[2]:

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\text{Groundwater head distribution of primary aquifers [m asl]}
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\[ q_{i,i+1} = \frac{0.73 \times 10^6 (\varphi_i - \varphi_{i+1}) k_i}{(m_i + m_{i+1}) k_i / k_{i+1}} \]  

where the following units are applied: \( \varphi \) [metres asl]; \( k \) [metres/day]; \( m \) [metres].

In (4), the vertical conductivity \( a_{z} = a_{i,i+1} \) of (2) is the mean harmonic of the vertical conductivities \( a_i \) and \( a_{i+1} \) of the \( i \)-th and \( (i+1) \)-th layers, accordingly:

\[ a_{i,i+1} = \frac{2 (a_i a_{i+1})}{a_i + a_{i+1}} \, , \quad a_i = 2 \frac{h^2 k_i}{m_i} \, , \quad a_{i+1} = 2 \frac{h^2 k_{i+1}}{m_{i+1}} \, . \]  

It follows from (4) and (5) that \( q_{i,i+1} \), as the infiltration flow for the \( (i+1) \)-th aquifer, is related to the surface \( z_{i+0.5} \), because the flow is controlled mainly by the conductivity \( a_i \) of the \( i \)-th aquitard. The subvector \( \varphi_{i+1} \) for the \( (i+1) \)-th aquifer is related to the surface \( z_{i+0.5} \).

The task of creating groundwater head and infiltration flow distributions for profiles is more complicated. To obtain correct results for the profiles, the new method of interpolation has been developed by scientists of RTU.

3. INTERPOLATION FOR GEOLOGICAL PROFILES

The map of geological profile assembles information (geological stratification, isoline distributions of groundwater head and infiltration flow, etc.) carried by vertical incisions of the HM body. The geological profile stratification is the geometrical interpolation on the set of \( z_i \) – surfaces.
The initial data of the subvectors \( \phi_i \) and \( q_{i,i+1} \) of the \( i \)-th geological layer for the profile can be obtained by interpolation on the corresponding grid files of these data on the intersection lines of the profile with the surface \( z_{i,0.5} \).

To create the profile \( \varphi \)-map, the interpolated \( \varphi \)-data located on the middle lines of aquitards and aquifers, as the intersections with surfaces \( z_{i,0.5} \), can be applied as boundary conditions \( \psi \) for the Laplace’s equation [7]. Its finite difference solution on the \( lz \)-plane of the profile represents its \( \varphi \)-map. The above approach is mostly applied.

In Fig. 4, the \( \varphi \)-map created by this seemingly right method is shown for the 2W–2E profile that has been used in [5] and [6]. The arbitrary \( lz \)-grid of the size 190×500 was applied; the grid steps: \( h_l = 1000 \) metres, \( h_z = 1.0 \) metre were used to approximate the profile longitude \( l \) and height \( z \), respectively. The solution (\( \varphi \)-map of Fig. 4) of this subsidiary problem was provided by the GV system. This \( \varphi \)-map is false, as the hydrogeological representation of groundwater processes for a profile. In nature, the head difference between the top and bottom of an aquifer is very small and any \( \varphi \)-isoline there should be vertical. This fact was not accounted for when the boundary conditions \( \psi \) were prepared for the \( \varphi \)-map of Fig. 4. To apply them rightly, it must be found out how the head difference \( \varphi_i - \varphi_{i+1} = \Delta_{i,i+1} = \Delta_i + \Delta_{i+1} \) distributes between the vertical conductances \( a_i \) and \( a_{i+1} \), if \( k_{i+1} >> k_i \):  

\[
\Delta_i = \Delta_{i,i+1} \frac{a_{i+1}}{a_i} \rightarrow \Delta_{i,i+1} ; \quad \Delta_{i+1} = \Delta_{i,i+1} \frac{a_i}{a_{i+1}} \rightarrow 0 .
\]

It follows from the estimate (6) that, to keep \( \Delta_{i+1} \approx 0 \), the top and bottom surfaces \( z_i \) and \( z_{i+1} \) of the \((i+1)\)-th aquifer, both must be used for fixing the conditions \( \psi_i = \psi_{i+1} = \varphi_{i+1} \). It is not necessary to use any special boundary conditions for the \( i \)-th aquitard, because
on its top and bottom surfaces the $\psi$ - conditions for the neighbouring aquifers are set. In Fig. 5, the correct $\varphi$–map for the 2W-2E profile is shown. It was obtained by GV when the estimates (6) were accounted for.

It is evident from the $\varphi$–map of Fig. 5 that, on borderlines parting aquitards from aquifers, isolines abruptly turn in verticals, within aquifers. It depends on choice of the step $h_z$, how strictly the break points of the isolines coincide with these borderlines (intersections with the $z$-surfaces).

To apply the SURFER option for showing directions of groundwater flows, the longitudinal gradient of the $\varphi$–map must be fictitiously increased, at least 250 times. If $h_l = 1000$ then the arrows of groundwater flow directions are vertical. When, for the grid file of $\varphi$, $h_l = 4.0$ is applied then the arrows take correct directions and the image of arrows must be joined with the $\varphi$–map by using the initial $h_l = 1000$.

To create distributions of infiltration flows for profiles, one have to assume that, within aquitards, isolines of infiltration must be verticals and the infiltration redistributes within aquifers. To confirm practical applicability of these suppositions, one must estimate the ratio $c = \frac{a_{xy}}{a_z}$ for the grid nodes of aquitards and aquifers:

$$c = \left(\frac{m_0}{h}\right)^2 \quad \text{for aquitards,}$$

$$c = \left(\frac{k}{k_0}\right) \left(\frac{m_0}{h}\right)^2 \quad \text{for aquifers} \quad (7)$$

where $k_0, m_0$ and $k, m$ are parameters of aquitards and aquifers, accordingly. It follows from (7) that the ratio $c$ is very small for the aquitards and its value may be considerable.

Fig. 5. Correct result for the groundwater head isolines of profile 2W-2E
for aquifers. Therefore, within aquitards in the profiles for infiltration flows, the isolines must be vertical, because there practically no change of the flows is possible.

In Fig. 6, the infiltration flow distribution \( q \)-map for the profile 2W-2E is shown. Together with the isolines, the SURFER color fill mode is applied.

By comparing the \( \varphi \)-maps of Fig.2, Fig. 5 and the \( q \)-maps of Fig. 3, Fig. 6 one can consider how the \( xy \)-data of the preQ aquifers are represented in the profile maps.

![Fig.6. Map for infiltration flow of profile 2W-2E](image)

To obtain valid \( q \) and \( \varphi \)-maps for profiles, one must keep \( h_l/h_z > 999 \). Then the quasi-solution of the Laplace’s equation gives vertical isolines, within aquitards and aquifers, accordingly. For the exact solution, when \( h_l/h_z = 1 \), the isolines are not verticales.

**4. USEFULLNESS OF THE GEOLOGICAL PROFILE MAPS**

Due to appliance of the profile \( \varphi \)-maps, necessity for important upgrades of the early LAMO version has been recognized. Two notable improvements have been accomplished:

- to avert unrealistic groundwater head distributions (shown for the profile 2W-2E in [5]), within the 24-th thick united aquifer D2ar#, the one has been split into its natural parts: the aquifers D2brt, D2ar and the aquitard D2arz (see Fig. 5.); the number of LAMO planes was increased from 25 to 27;
• valleys of rivers have been fully implemented into the HM body; for the early LAMO version (applied to prepare [5]), the valleys were immersed only into the Quaternary strata. Simulation of rivers with deep valleys, as very influential elements of HM, has been considerably improved.

The profile $q$–maps have helped to find out and to correct mistakes of LAMO, especially, the ones related to joining of rivers with the HM body.

5. CONCLUSIONS

The geological profile map assembles the $xy$-type data of HM planes into the $lz$-picture that is projected on the geological stratification formation. The map for a profile provides worthy information that cannot be easily deduced from the separate $xy$–maps which data are included in the profile.

The interpolation method has been developed for creating isoline maps of groundwater head and infiltration flows for profiles.

Due to use of geological profile maps, important upgrading of LAMO has been initiated. These maps have helped to gain new knowledge and better understanding of the complicated groundwater processes of Latvia that are simulated by LAMO.

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REFERENCES


